1. (12 %) What's the value of

$$\frac{1}{2\pi i}\int_C \frac{3z^2+7z+1}{z+1}$$

where C is the circle $|z+1|=1$? What is the value of this integral if C is the circle $|z|=\frac{1}{2}$?

2. (13 %) The oscillations of a mechanical system are governed by the equations

$$\begin{aligned} M_1\ddot{y}_1 &= -\lambda_1 y_1 + \lambda_2(y_2-y_1), \\ M_2\ddot{y}_2 &= -\lambda_2(y_2-y_1) + \lambda_3(y_3-y_2), \\ M_3\ddot{y}_3 &= -\lambda_3(y_3-y_2) - \lambda_4 y_3. \end{aligned}$$

Assuming the displacements y_i may be expressed in the form $y_i = x_i\cos\omega t (i=1,2,3)$, show that the equations may be written in the matrix form $(\mathbf{A}-\omega^2\mathbf{I})\mathbf{X}=\mathbf{0}$. if $M_1=1, M_2=2, M_3=3$, $\lambda_1=1, \lambda_2=1, \lambda_3=2, \lambda_4=3$, determine the natural frequencies and amplitude ratios of the system.

3. (13 %) The position vector to a curve C is assumed to be expressed in the form $\mathbf{r}=x(t)\mathbf{i}+y(t)\mathbf{j}+z(t)\mathbf{k}$, and a prime is used to denote differentiation with respect to the parameter t, whereas s denotes arc length along C and ρ denotes the radius of curvature. Show that

$$\frac{1}{\rho}\mathbf{n} = \frac{d\mathbf{u}}{ds} = \frac{\mathbf{u}'}{s'} = \frac{(\mathbf{r}'\cdot\mathbf{r}')\mathbf{r}''-(\mathbf{r}'\cdot\mathbf{r}'')\mathbf{r}'}{|\mathbf{r}'|^4} = \frac{(\mathbf{r}'\times\mathbf{r}'')\mathbf{r}'}{|\mathbf{r}'|^4}$$

where $\mathbf{u}$ is a unit vector tangent to the curve and $\mathbf{n}$ is a unit vector perpendicular to the tangent vector $\mathbf{u}$.

4. (12 %) Obtain the general solution of the differential equation in terms of *Maclaurin series*.

$$x\frac{d^2y}{dx^2}-\frac{dy}{dx}-4x^3y=0$$

5. (15%) The sampled function cos kw_0T represents samples of the waveform cos w_0t at intervals of T. It is desired that a recursive equation be found (difference equation) which will generate this function. Determine this difference equation and the proper initial conditions.

6. (15%) An implantable drug delivery system consists of a box that is filled with a homogeneous carrier material containing the drug. The height of the box is small compared with the length of the sides of the top and bottom plates, which are square. The top and bottom plates, and three sides are impermeable to the drug. The drug can diffuse out of the system into the body through the fourth side. Assume that the concentration is governed by the one-dimensional diffusion equation. At the time of implantation, the drug concentration is c_0 everywhere in the system. The concentration of the drug in the body is zero at the time of implantation and remains zero, since the body acts as a large sink.
(1). Find the drug concentration function c(x,t).
(2). Sketch the mass flow out of the box as a function of time.

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7. (10%) A sheet of metal coincides with the square in the xy-plane whose corners are the points (0,0), (1,0), (1,1), (0,1). The two faces are perfectly insulated and the sheet is so thin that heat flows in it can be regarded as two-dimensional. The edges parallel to the x-axis are perfectly insulated and the left hand edge is kept at the constant temperature $u=0$. If the temperature distribution $u(1,y)=f(y)$ is maintained along the right hand edge. Find the steady state temperature distribution function $u(x,t)$ throughout the sheet.

8. (10%) The difference equation for nitrogen washout is given by

$$x_{k+1} = x_k * r + (1-r) \quad \text{with } x_0 = 0.2, \text{ and } r \text{ is a constant.}$$

Solve for x_k.