(甲.乙)

國立成功大學八十一學年度醫学工程研究所入學考試(工程數學試題)	共 1 頁 第 1 頁

1. **(15%)** Solve the differential equation with initial conditions $y(0) = 0$ and $y'(0) = 0$

$$\frac{d^2y(t)}{dt^2} + 4\frac{dy(t)}{dt} + 3y(t) = f(t)$$

where the function $f(t)$ is shown in Fig. 1. (Hint: Using Laplace transformation)

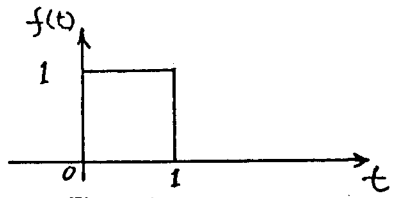


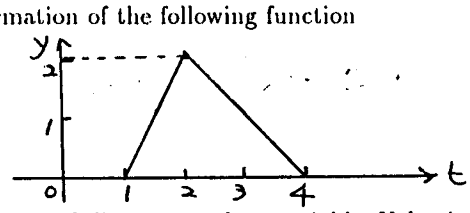
Figure 1:

2. **(10%)** Let the matrix

$$A = \begin{bmatrix} 1 & -4 \\ 1 & 1 \end{bmatrix}$$

Find the eigenvalues, eigenvectors and its inverse matrix.

3. **(15%)** Find the Laplace transformation of the following function



4. **(10%)** Find the mean and variance of discrete random variable X having the probability function $f(-1) = 0.1$, $f(0) = 0.1$, $f(1) = 0.7$ and $f(2) = 0.1$.

5. **(12%)** Evaluate

$$I = \int_0^{2\pi} \frac{d\theta}{3 - 2\cos\theta}$$

6. **(13%)** Show that $u(x, y) = x(1-y)$ is harmonic for all x and y and find the conjugate function $v(x, y)$. Find the function $f(z) = u + iv$ and write it in terms of z.

7. **(12%)** Suppose that $\Phi(x, y, z)$ satisfies Laplace's equation $\nabla^2\Phi = 0$ everywhere within a region $\Re$. Show that the flux of grad Φ from $\Re$ vanishes.

8. **(13%)** Determine the surface area of the paraboloidal shell given by

$$z = x^2 + y^2, \quad 0 \le z \le 1.$$

If the mass per unit area of the shell is proportional to the height z, determine the total mass of the shell.

Some Functions $f(t)$ and Their Laplace Transforms $\mathcal{L}(f)$

	$f(t)$	$\mathcal{L}(f)$		$f(t)$	$\mathcal{L}(f)$
1	1	$1/s$	6	e^{at}	$\frac{1}{s-a}$
2	t	$1/s^2$	7	$\cos \omega t$	$\frac{s}{s^2+\omega^2}$
3	t^2	$2!/s^3$	8	$\sin \omega t$	$\frac{\omega}{s^2+\omega^2}$
4	t^n $(n = 1, 2, \cdots)$	$\frac{n!}{s^{n+1}}$	9	$\cosh at$	$\frac{s}{s^2-a^2}$
5	t^a (a positive)	$\frac{\Gamma(a+1)}{s^{a+1}}$	10	$\sinh at$	$\frac{a}{s^2-a^2}$