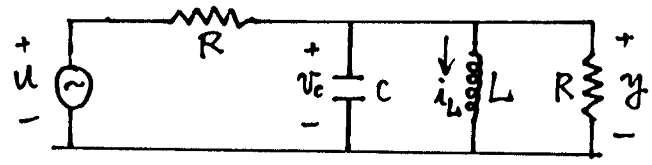
(15%) 1. The circuit shown below has u as input and y as output

(7%) (a) Using $x_1 = v_c$ and $x_2 = i_L$ as state variables, find a state-space model.

(8%) (b) Find the transfer function from u to y.



(25%) 2. The system is described as state equation

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 2 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

(5%) (a) Test the system stability ? controllability ? and observability ?

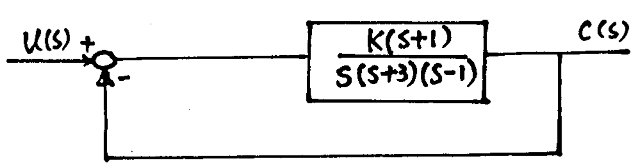
(5%) (b) If the system is unstable, can you stablize the system through state feedback? why?

(5%) (c) Can you assign both of the eigenvalues to -3 by state feedback ? why?

(5%) (d) Find the transfer function of the system.

(5%) (e) Compare the order of (d) with that of above system. equal? or not equal? why?

(20%) 3. For an unstable plant, Mr. Wang arranges the system as following,



(7%) (a) Find the ranges of K, let the system be stable forever.

(13%) (b) Plot a root locus for the system, label the important points.

(20%) 4. The loop transfer function of a single-loop feedback control system is

$$G(s)\,H(s) = \frac{K}{s(s+1)}$$

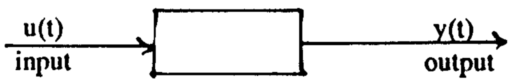
where K is a positive constant.

(10%) (a) Sketch the Nyquist plot and check the system stability.

(5%) (b) Determine the value of K so that the gain margin of the system is 20 dB.

(5%) (c) Determine the value of K so that the phase margin of the system is 60°.

(20%) 5. For each of the following systems, determine whether or not the system is (i) Linear, (ii) Causal, (iii) Time-invariant (Give your reasons)



(5%) (a) $y(t) = \int_0^t u(\tau)\, d\tau \quad , t \geq 0$

(5%) (b) $y(t) = \int_t^{t+1} u(\tau-2)\, d\tau$

(5%) (c) $y(t) = \mathrm{sgn}[u(t-1)]$

$$\text{where } \mathrm{sgn}[x] = \begin{cases} 1, & \text{if } x \geq 0 \\ -1, & \text{if } x < 0 \end{cases}$$

(5%) (d) $y(t) = \int_{-\infty}^{\infty} g(t-\tau)\, u(\tau)\, d\tau$, where

$$g(t) = 2\omega \frac{\sin 2\omega(t-t_0)}{2\omega(t-t_0)} \; ; \; \omega \text{ and } t_0 \text{ are constants}$$