1. (15 %) A skew-symmetric matrix $\tilde{\mathbf{a}}$ associated with an algebraic vector $\mathbf{a} = [a_x, a_y, a_z]^T$ is defined as

$$\tilde{\mathbf{a}} \equiv \begin{bmatrix} 0 & -a_z & a_y \\ a_z & 0 & -a_x \\ -a_y & a_x & 0 \end{bmatrix}$$

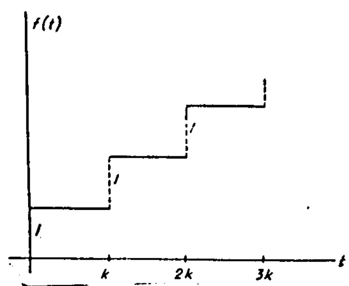
The vector product $\vec{c} = \vec{a} \times \vec{b}$, can thus be written in algebraic vector form as

$$\mathbf{c} = \tilde{\mathbf{a}}\mathbf{b} = \begin{bmatrix} a_y b_z - a_z b_y \\ a_z b_x - a_x b_z \\ a_x b_y - a_y b_x \end{bmatrix}$$

Proof that: $\widetilde{(\tilde{\mathbf{a}}\mathbf{b})} = \mathbf{b}\mathbf{a}^T - \mathbf{a}\mathbf{b}^T = \tilde{\mathbf{a}}\tilde{\mathbf{b}} - \tilde{\mathbf{b}}\tilde{\mathbf{a}} \Rightarrow \tilde{\mathbf{a}}\tilde{\mathbf{b}} + \mathbf{a}\mathbf{b}^T = \tilde{\mathbf{b}}\tilde{\mathbf{a}} + \mathbf{b}\mathbf{a}^T$

2. (20%) What is the Laplace transform of the staircase function

$$f(t) = n+1, \quad nk < t < (n+1)k \quad n = 0, 1, 2, \ldots$$



3. (15%) Proof that

$$\int_C \phi \, d\mathbf{R} = \iint_S \mathbf{N} \times \nabla\phi \, dS$$

where ϕ is a scalar point function; **R** is the vector from the origin to a general pint on the surface S; **N** is a unit normal; and C is the closed curve bounding the surface S.

4. (20%) Find the following integral:

(a) $\int_0^{2\pi} \frac{d\theta}{\sqrt{2} - \cos\theta}$ (b) $\int_0^{\infty} \frac{dx}{1+x^4}$

5. (15%) Suppose X and Y have joint density

$$f_{XY}(x,y) = \begin{cases} (1+xy)c & ,2 \le x \le 3 \quad ,1 \le y \le 2 \\ 0, \textit{otherwise} \end{cases}$$

(a) Find c , (b) Find f_X and f_Y

6. (15%) Prove the following statement: Let λ_1 and λ_2 be distinct eigenvalues of a matrix A, then the corresponding eigenvectors x_1 and x_2 are linearly independent.