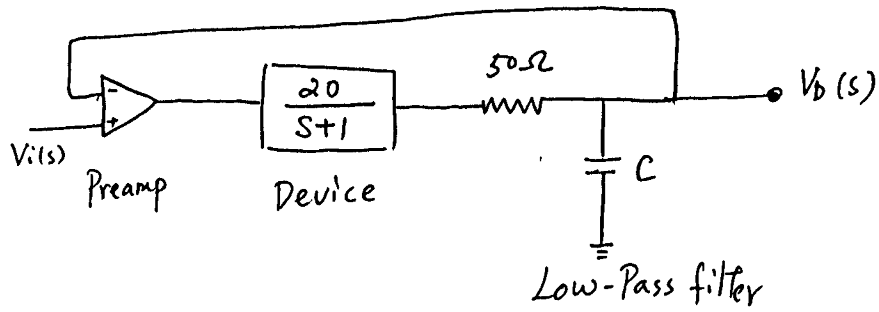
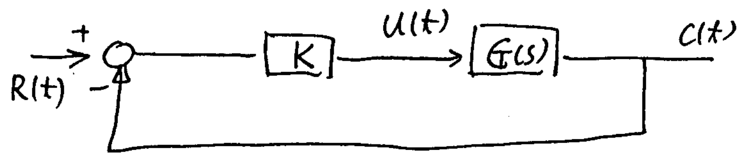
20% 1. A device with low output impedance is shown below in cascade with a low pass filter and a preamplifier. The amplifier has high input impedance with a gain of one and is used for adding the signals as shown. Describe the procedure to select a value of capacitor so that the transfer function $V_o(s) / V_i(s)$ has a damping ratio of 0.707. and finding the transfer function.



20% 2. A system is described as $\dot{x}(t) = A\,x(t) + b\,u(t)$. We can use the state feedback $u(t)=k\,x(t) + r(t)$ to assign the eigenvalues of the system arbitrarily. (a)Prove the property of controllability does not change under state feedback. (that is (A,b) is controllable if only if (A+bk,b) is controllable.(b) Is this still true in observality through state feedback? explain your reasons!

30% 3. Given the following feedback control system, where the plant dynamics are described by the following differential equation:

$$\dddot{c}(t) + 7\,\ddot{c}(t) + 10\,\dot{c}(t) = u(t)$$



a. Find G (s)

b. Plot the locus of roots of the closed-loop system as K is varied from 0 to ∞ .

c. Find the range of K for which the closed loop is stable.

d. If one of the closed-loop roots is at s = - 6, for K=24, find the other two roots.

e. Using the dominant roots determined in part d , find the damping ratio .

30% 4. Consider the linear system

$$\dot{x}(t) = A\,x(t) + B\,u(t) \qquad \text{where } A = \begin{bmatrix} 3 & 0 \\ 1 & 4 \end{bmatrix}$$

$$y(t) = C\,x(t) + D\,u(t) \qquad \text{and } D = [\,1\,].$$

a. If $B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $C = [\,1 \;\; 1\,]$ find the system's transfer function T(s) in its simplest form.

b. (True or False) The system is stable.
(True or False) The system is controllable.
(True or False) The system is observable.

c. An input-output differential equation for this system can be written in the form:

$$\dot{y}(t) + \alpha_1 \; y(t) = \alpha_2 \;\; \dot{u}(t) + \alpha_3 \;\; u(t)$$

Find $\alpha_1 =$ $\qquad \alpha_2 =$ $\qquad \alpha_3 =$

d. Compute the exponential matrix e^{At} for this system.

e. If $e^{At} = Z_1 e^{\lambda_1 t} + Z_2 \;\; e^{\lambda_2 t} \, e^{At} = Z_1 e^{\lambda_1 t} + Z_2 e^{\lambda_2 t}$, find the matrices Z_1 and Z_2 and the eigenvalues.

f. Does the impulse response completely specify the dynamics of the state model? Why or Why not?