

1. Evaluate the following surface integral

$$I = \iint_S (x dy dz + y dx dz + z dx dy)$$

Where S is the closed surface consisting of the cylinder $x^2 + y^2 = a^2$ ($0 \leq z \leq b$) and the circular disk $z = 0$ and $z = b$ ($x^2 + y^2 \leq a^2$). Hint: Divergence theorem of Gauss. (10 points)

2. For a matrix A,

(I) Describe the following terminologies. (3 points each)

- (a) Rank
- (b) Nullity
- (c) Eigenvalue
- (d) Eigenvector
- (e) Generalized eigenvector

(II) Can we find a nonsingular matrix T such that $T^{-1}AT$ becomes a diagonal matrix for any matrix A? Why? (5 points)

(III) For $A = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$, find the three eigenvectors and verify they are linearly independent. (5 points)

3. Find the following integral

(a) $\int_{-\infty}^{\infty} \frac{\cos 3x}{1+x^2} dx$ (8 points) (b) $\int_0^{2\pi} \frac{d\theta}{2+\cos \theta}$ (7 points)

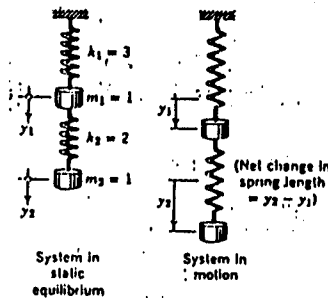
4. Find the analytical solution of the vertical vibration mechanical system as in the figure below which governed by the system of differential equations. (20 points)

$$\dot{Y} = A Y$$

$$\text{where } Y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}, \quad A = \begin{bmatrix} -5 & 2 \\ 2 & -2 \end{bmatrix}$$

with the initial conditions

$$y_1(0) = 1, \quad y_2(0) = 2, \quad \dot{y}_1(0) = -2\sqrt{6}, \quad \dot{y}_2(0) = \sqrt{6}$$



5. Write down a 2 dimensional Laplace's equation and what is the type of this linear partial differential equation? (Hint: parabolic etc.) (10 points)
6. What is the ill-conditioned of a system of linear equations when solving numerically? give an example. (10 points)
7. When testing the hypothesis of a static analysis, it is necessary to have a number α which is called the significance level and it is customary to choose $\alpha = 1\%$ or 5% , what is the meaning of the value of this significance level? (Hint type-I error)(10 points)