1. Evaluate the following surface integral

$$I = \iint_S (x^3 dydz + x^2 ydxdz + x^2 zdxdy)$$

Where S is the surface consisting of the cylinder $x^2 + y^2 = a^2$ $(0 \le z \le b)$ and the circular disk $z = 0$ and without disk $z = b$ $(x^2 + y^2 \le a^2)$. That is, the surface like a sugar can without the upper cover.
(Hint: Using Divergence thorem of Gauss. However, the surface S is not a closed surface. Be careful!)(20 points)

2. For a 2 x 2 matrix A,
   (a) Find a matrix A, let the eigenvectors are orthogonal.(5 points)
   (b) Find A such that y = Ax is a counterclockwise rotation through 30° in the plane.(5 points)
   (c) Show the eigenvectors associated with distinct eigenvalues are linearly independent.(5 points)

3. Describe the following terminologies;(3 points each)
   (a) Simple curve.
   (b) Simple connected domain or Simple connected region.
   (c) Analytic function
   (d) Stoke's theorem
   (e) Directional derivative

4. (15%) Solve two unknowns x_1 and x_2 in two simultaneous equations as below

$$\dot{x}_1 + 2x_1 - 2\dot{x}_2 + 3x_2 = 4$$
$$2\dot{x}_1 + x_1 + \dot{x}_2 - x_2 = 2t$$

when $t = 0^+$, $x_1 = 1$, $x_2 = -2$.

5. (15%) What is the *Residue Theorem*? Evaluate the definite integrals $\int_{-\infty}^{\infty} \frac{x^2 dx}{1 + x^6}$ by the method of residues.

6. (20%) If $z = z(y, t)$ is a displacement, the model to describe the vibration of a flexible string of length l is as follows

$$z_{tt} = c^2 z_{yy},\ 0 < y < l,\ 0 < t$$
$$z_y(0, t) = 0,\ z_y(l, t) = 0,\ 0 < t$$
$$z(y, 0) = z_0(y),\ z_t(y, 0) = 0,\ 0 \le y \le l$$

The string supports are of the free-boundary type (lateral, but no vertical restraint), and no external forces are acting. Solve the differential equation by separation of variables.