**5. Mathematical Statistics:**

(a) *Central Limit Theorem* is the most important/fundamental theory for statistics application in Biology, Medicine and Health Sciences. Please describe the Central Limit Theorem and explain their application in Biostatistics. (10%)

(b) The following displacement-voltage data were obtained from calibration testing of a new sensor design. Use least-squares error method for linear regression analysis of the data. (10%)

| Displacement (mm) | 5, | 10, | 15, | 20, | 25, | 30, | 35, | 40, | 45, | 50 |
|---|---|---|---|---|---|---|---|---|---|---|
| Output Voltage (V) | 0.52, | 0.99, | 1.50, | 1.98, | 2.61, | 2.98, | 3.47, | 4.1, | 4.52, | 4.97 |

**6. Complex Analysis:**

(a) {use Cauchy's integral formula and partial fractions} Integrate $g(z) = \left(z^2 - 1\right)^{-1} \cdot \tan z$ around the circle C: $|z| = 3/2$ (counterclockwise) (10%)

(b) Find the linear fractional transformation ($w = \dfrac{az + b}{cz + d}$ , $ad - bc \neq 0$) that maps $z_1 = -1$, $z_2 = i$, $z_3 = 1$, onto $w_1=0$, $w_2=i$, $w_3=\infty$ and make a sketch of the disk and the half-plane. (10%)

**7. Numerical Methods:**

Interpolate $f_0 = f(0)=1$, $f_1 = f(2)=9$, $f_2 = f(4)=41$, $f_3 = f(6)=41$ by the cubic spline satisfying $k_0=0$, $k_3 = -12$. (20%)

Given the following equations:

$$k_{j-1} + 4k_j + k_{j+1} = \frac{3}{h}(f_{j+1} - f_{j-1}), \quad j = 1, \cdots, n-1,$$

$$p_j(x) = a_{j0} + a_{j1}\left(x - x_j\right) + a_{j2}(x - x_j)^2 + a_{j3}(x - x_j)^3 .$$

$$a_{j0} = p(x_j) = f_j$$

$$a_{j1} = p_j'(x_j) = k_j$$

$$a_{j2} = \frac{1}{2}p_j''(x_j) = \frac{3}{h^2}(f_{j+1} - f_j) - \frac{1}{h}(k_{j+1} + 2k_j)$$

$$a_{j3} = \frac{1}{6}p_j'''(x_j) = \frac{2}{h^3}\left(f_j - f_{j+1}\right) + \frac{1}{h^2}(k_{j+1} + k_j)$$