

請於下列六題中任選五題作答 (Take 5 from the following 6 questions)

1. System of Differential Equations, Phase Plane and Stability

A damped pendulum consists of a body of mass M , a dashpot of damping constant C and a rod of length L . Setting up the mathematical model and determine the locations and types of the critical points. Assume that the mass of the rod and air resistance are negligible. (20 points)

2. Partial Differential Equations

Find the vibrations of a rectangular membrane of sides $A = 4$ ft and $B = 2$ ft. if the tension is 12.5 lb/ft, the density is 2.5 slugs/ft², the initial velocity is 0, and initial displacement is $F(x, y) = 0.1(4x - x^2)(2y - y^2)$ ft, where $0 \leq x \leq 4$, $0 \leq y \leq 2$. **Hint:** Use Double Fourier series to solve a vibrating membrane. (20 points)

3. Linear Algebra and Matrices

(a) Use Gauss Elimination to solve the following linear system: (10 points)

$$\begin{aligned} x_1 - x_2 + x_3 &= 0 \\ -x_1 + x_2 - x_3 &= 0 \\ 10x_2 + 25x_3 &= 90 \\ 20x_1 + 10x_2 &= 80 \end{aligned}$$

(b) Find a symmetric matrix C such that $Q = x^T C x$, and prove that matrix C is positive definite (10 points), where $Q = -3x_1^2 + 4x_1x_2 - x_2^2 + 2x_1x_3 - 5x_3^2$

4. Vector Differential and Integral Calculus

(a) If the potential between two concentric cylinders is $V(x, y) = 110 + 30 \ln(x^2 + y^2)$ [volts], what is the direction of the electric force at point $P(x=2, y=5)$? Show that $\nabla^2 V = 0$ (10 points)

(b) Find the work done by the force $F = (2xy^3 \sin z, 3x^2y^2 \sin z, x^2y^3 \cos z)$ in the displacement around the curve of intersection of the paraboloid $z = x^2 + y^2$ and the cylinder $(x - 1)^2 + y^2 = 1$. (10 points)

5. Probability and Mathematical Statistics

- (a) A motor drives an electric generator. During a 30-day period, the motor needs repair with probability 8% and the generator needs repair with probability 4%. What is the probability that during a given period, the entire apparatus will need repair? (7 points)
- (b) The following sample data were from 32 scores of a Biostatistics test. Is the sample data taken from a normal distribution? Please describe statistical principles and procedures for the data analysis even without tedious computation. (13 points)

Test Scores

82	98	77	41	67	55	90	83	72	86	60	65	79	85	75	70
58	65	87	81	74	72	66	90	84	78	63	69	76	55	81	58

6. Complex Analysis

- (a) Map the angular region $D: -\pi/6 < \arg z < \pi/6$ onto the unit disk $|w| \leq 1$. (10 points)
- (b) Integrate the function $f(z) = z^{-4} \sin z$ counterclockwise around the unit circle C . (10 points).