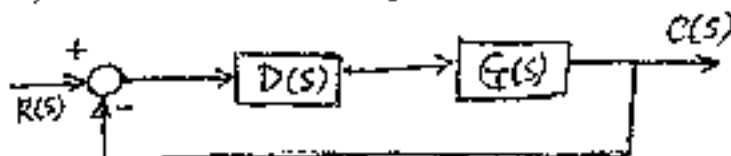


1. (35%) 如果一系統的 transfer function 為

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

- (a) 試求該系統的 Impulse response 為何? (6 points)
 (b) 如果輸入為 unit step 函數, 則輸出為何? (6 points)
 (c) 如果輸入為 unit step 函數, 那麼該系統的 Maximum overshoot 為何?
 (6 points)
 (d) 何種條件下該系統為 BIBO (Bounded Input Bounded Output) stable?
 (6 points)
 (e) 如果該系統是一 filter 它應該是一個 low-pass filter 或是 high-pass filter? 說明之。 (6 points)
 (f) 有一個二階系統為 $\frac{1}{(s+100)(s+5)}$, 如果要將它以一階系統來近似應該為 $\frac{1}{s+100}$ 或者為 $\frac{1}{s+5}$? 說明之。 (5 points)

2. (45%) Consider the feedback control system



with $G(s) = \frac{1}{s(s+1)}$

- I. Suppose the design of this system requires the undamped natural frequency, $\omega_n = 2$; find K using D(s) as a proportional control, i. e., $D(s) = K$
- (a) What are the closed-loop eigenvalues of the above system for this value of K? (5 points)
 (b) Is Overshoot greater than 15%? (5 points)
 (c) Calculate the steady state error for unit ramp input. (5 points)
- II. Suppose the design specifications require dominant closed loop roots with $\omega_n = 2$ and $\zeta > 0.5$ and steady state error of 0.1 for a ramp input. To satisfy this, the following form is chosen for D(s)

$$D(s) = K \frac{Ts+1}{\alpha Ts+1}$$

where $\alpha = 0.2$ and $T = 0.5$ are chosen to satisfy the transient response criteria.

- (a) Is this compensation called "Integral control", "Lead compensation" or "Lag compensation"? (5 points)
 (b) Complete the design by choosing K to satisfy the steady state requirement (5 points)

(背面仍有題目, 請繼續作答)

- III. (a) Sketch the frequency response of $D(s)G(s)$ obtained in part II.
 (Bode plot or Nyquist plot). (10 points)
 (b) Find the gain crossover frequency? (5 points)
 (c) Find the phase margin, PM. (5 points)

3. (20%)

- a. An analog feedback control system is designed to limit the overshoot to 10% for a step input. The gain of the system is set so that the feedback system has the following closed-loop roots.

$$-5 \pm j5, -10$$

Does the system satisfy the required specification? Explain. (10 points)

- b. A digital control system has the following closed-loop transfer function

$$\frac{C(z)}{R(z)} = \frac{0.368z + 0.264}{z^2 - z + 0.632}$$

Is the closed-loop system stable? Explain. (10 points)

Laplace Transform	Time Function	s-Transform
1	Unit impulse $\delta(t)$	1
$\frac{1}{s}$	Unit step $u(t)$	$\frac{z}{z-1}$
$\frac{1}{s - \sigma}$	$f(t) = \sum_{n=0}^{\infty} a^n \delta(t - nT)$	$\frac{z}{z-1}$
$\frac{1}{s^2}$	t	$\frac{Tz}{(z-1)^2}$
$\frac{1}{s^2 + \omega^2}$	$\frac{t}{T}$	$\frac{T^2 z(z+1)}{2(z-1)^2}$
$\frac{1}{s^2 + \omega^2}$	$\frac{t^2}{2T}$	$\lim_{n \rightarrow 0} \frac{(-1)^n \frac{\partial^n}{\partial \omega^n} \left(\frac{z}{z - e^{-\omega T}} \right)}{n!}$
$\frac{1}{s + a}$	e^{-at}	$\frac{z}{z - e^{-aT}}$
$\frac{1}{s^2 + a^2}$	$t e^{-at}$	$\frac{T z e^{-aT}}{(z - e^{-aT})^2}$
$\frac{a}{s^2 + a^2}$	$1 - e^{-at}$	$\frac{(1 - e^{-aT})z}{(z-1)(z - e^{-aT})}$
$\frac{\omega}{s^2 + \omega^2}$	$\sin \omega t$	$\frac{z \sin \omega T}{z^2 - 2z \cos \omega T + 1}$
$\frac{\omega}{(s+a)^2 + \omega^2}$	$e^{-at} \sin \omega t$	$\frac{z e^{-aT} \sin \omega T}{z^2 - 2z \cos \omega T + 1}$
$\frac{s}{s^2 + \omega^2}$	$\cos \omega t$	$\frac{z(z - \cos \omega T)}{z^2 - 2z \cos \omega T + 1}$
$\frac{s+a}{(s+a)^2 + \omega^2}$	$e^{-at} \cos \omega t$	$\frac{z^2 - z e^{-aT} \cos \omega T}{z^2 - 2z e^{-aT} \cos \omega T + e^{-2aT}}$