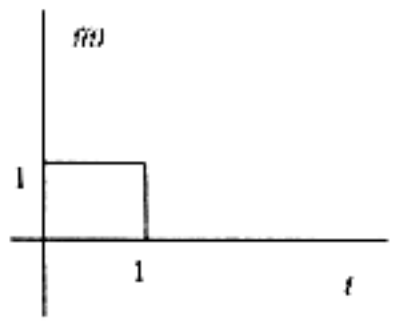
1. (20 points)Solve the following equations: (10 points each)

(a) $y''(t) + 3y'(t) + 2y(t) = f(t)$ $\quad y'(0) = 0,\ y(0) = 0$

when



(b) $y''(t) + \omega_0 y(t) = \sin \varpi t$ $\quad y'(0) = 0, y(0) = 0,$ $\quad \varpi_0 \neq \varpi$

2. (20points)For a matrix $A = \begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix}$, (10 points each)

(a) Find its inverse matrix.

(b) Can you find a nonsingular matrix Q. such that $Q^{-1}AQ$ is a diagonal matrix?

3. (20 points)For an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, (10 points each)

(a) Derive the area of ellipse equal to πab.

(b) If a uniform ellipse plate with total mass M, find the moment of inertia about the z-axis at the center of ellipse.

4. (10 points)Evaluate the improper integral, $\int_0^\infty \frac{dx}{1+x^4}$

5. (10 points) For an unknown 3x3 matrix, the mapping relationship as follows

vector (3, 2, 1) → vector (6, 5, 4)

vector (6, 5, 4) → vector (9, 8, 7)

That is $\begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}\begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 6 \\ 5 \\ 4 \end{bmatrix}, \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}\begin{bmatrix} 6 \\ 5 \\ 4 \end{bmatrix} = \begin{bmatrix} 9 \\ 8 \\ 7 \end{bmatrix}$

Please find $\begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}\begin{bmatrix} 9 \\ 8 \\ 7 \end{bmatrix} = ?$ without the information of the matrix.

6. (20 points)Please explain the following theorems (5 points each)

(a) Central limit theorem

(b) Green's theorem

(c) Divergence theorem of Gauss

(d) Residue theorem