1. (10%) Ten randomly selected nails had the lengths:
0.80, 0.81, 0.82, 0.81, 0.80, 0.81, 0.82, 0.81, 0.81, 0.82.
Find the sample mean and sample variance for these ten nails.
(Hints: $\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$, $s^2 = \frac{1}{n-1}\sum_{j=1}^{n}(x_j - \bar{x})^2$)

2. (20%) Newton's Method is an iteration numerical method for solving equation $f(x) = 0$, where $f(x)$ is differentiable. The general formula for Newton's iteration is

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

Find the results (up to 5 digits) of the first three Newton's iterations, i.e., x_1, x_2, x_3, of $f(x) = x^2 - 2 = 0$ with $x_0 = 1$. How could you justify that the iteration process (solution) is convergent?

3. (20%) Find a unit normal vector ***n*** of
(a)plane $4x+2y+4z = -7$ (8%)
(b)cone of revolution $z^2=4(x^2+y^2)$ at the point P:(1,0,2) (12%)
(Hint: using gradient)

4. (20%) An elastic membrane in the $x_1\, x_2$-plane with a boundary circle $x_1^2 + x_2^2 = 1$ is stretched so that a point P: (x_1, x_2) goes over into the point Q: (y_1, y_2) given by

$$y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = Ax = \begin{bmatrix} 5 & 3 \\ 3 & 5 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}.$$

Please find certain points (x_1, x_2) at the circle, such that the values of $y_1^2 + y_2^2$ is maximum.

5. (10%) Show that the distinct eigenvalues associated eigenvectors are linearly independent.

6. (20%) Please find the (a) Laplace transform (b) Fourier series of the periodic function $f(t)$ in figure 1.

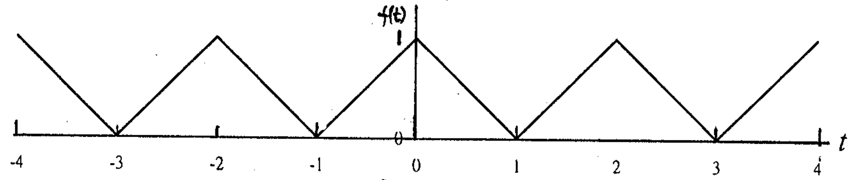

figure 1