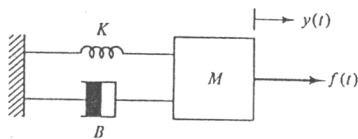


1. (30%) For a mechanical system as following,



M: mass, K: spring constant, B: damping constant, $f(t)$: applied force, y : displacement.

- Write down the differential equation of the system. (3 points)
- Describe this system by dynamic equation. (3 points)
- From the energy point of view, explain these three components on the system. At the same time, point out M, B and K are analogous to RLC network. R: resistor, L: inductor, C: capacitor (5 points)
- Derive the transfer function of the system. (4 points)
- If $B=0$, derive and plot the impulse response of the system. (7 points)
- If $B=0$, $f(t) = \cos \omega_0 t$ and $\omega_0 = \sqrt{\frac{K}{M}}$ then find and plot the response of the system. (8 points)

2. (25%) Consider the following state equation and output equation

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

- Find the transfer function. (5 points)
- Can you check the controllability from above transfer function? Why? (5 points)
- Check the stability of the system. (5 points)
- Design a state feedback $u = -Gx + r$ to stabilize the system. (Hint: change any positive eigenvalue to negative eigenvalue) (10 points)

3. (15%) Show three types of common nonlinearities encountered in control system.

4. (30%) The open-loop transfer function of a unity feedback control system is

$$G(s) = \frac{K}{s(1+0.1s)(1+s)}$$

- (a) Determine the value of K so that the resonance peak M_p of the system is equal to 1.4. (7 points)
- (b) Determine the value of K so that the gain margin of the system is 20 dB. (7 points)
- (c) Determine the value of K so that the phase margin of the system is 60° . (7 points)
- (d) Find ω_p , resonant frequency in (a), ω_c , phase crossover frequency in (b) and ω_g , gain crossover frequency in (c). (9 points)