

1. (20 points) First- and Second-Order Differential Equations (10 points each)

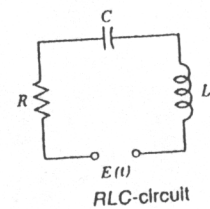
- (a) A tank contains 1000 gal of water in which 200 lb of salt are dissolved. Fifty gallons of brine, each containing $(1 + \cos t)$ lb of dissolved salt, run into the tank per minute. The mixture, kept uniform by stirring, runs out at the same rate. Find the amount of salt $y(t)$ in the tank at any time t .
- (b) Find the steady-state current in RLC -circuit when $R=50$ ohms, $L=30$ henrys, $C=0.025$ farad, $E(t)=200 \sin 4t$ volts. (參考圖 -)

2. (20 points) Laplace Transforms and Fourier Analysis (10 points each)

- (a) Please state the reason why an engineer needs to learn and understand Laplace and Fourier transformation, Fourier series and Integrals.
- (b) For a full-wave rectification of $\sin wt$, find its Laplace transform and Fourier Series representation.

3. (20 points) Probability and Mathematical Statistics (10 points each)

- (a) Please explain "Normal or Gauss Distribution" by using descriptive and quantitative statements.
- (b) Compute the probability of obtaining at least two "six" in rolling a fair die 4 times.

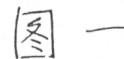


4. (20 points) Linear Algebra (10 points each)

- (a) Find the inverse of the matrix $[A]$
- (b) Find the eigenvalues and eigenvectors of the matrix $[B]$

where:

$$[A] = \begin{bmatrix} 3 & 1 \\ 2 & 4 \end{bmatrix} \quad [B] = \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$$



5. (20 points) Numerical Analysis

- What is (a) **ill-condition** of a linear system equations? (7%)
- (b) **convergence** of an iteration process? (7%)
- (c) **spline** in function approximation? (6%)