Part I. Linear Algebra (50%)

1. Find the best quadratic least squares fit to the data (15%)

x	0	1	2	3
y	3	2	4	4

2. Let $A = \begin{bmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{bmatrix}$

   (a) Compute the LU factorization of A. (10%)

   (b) Check whether A is positive definite or not. (5%)

3. The set $S = \left\{ \frac{1}{\sqrt{2}}, \cos x, \cos 2x, \cos 3x, \cos 4x \right\}$ is an orthonormal set of vectors in space $C[-\pi, \pi]$ with inner product $< f, g >= \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) g(x) dx$.

   (a) Use trigonometric identities to write the function $\sin^4 x$ as a linear combination of elements of S. (10%)

   (b) Find the values of the integrals $\int_{-\pi}^{\pi} \sin^4 x \cos x \, dx$ and $\int_{-\pi}^{\pi} \sin^4 x \cos 4x \, dx$. (10%)

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1. [15%]

Define $\delta_n = \begin{cases} 1 & ,n=0 \\ 0 & ,otherwise \end{cases}$ and $u_n = \begin{cases} 0 & ,n<0 \\ 1 & ,otherwise \end{cases}$, for all integer n.

(a) Let $y_n = 0$, for $n<0$. Define a recurrence relation as

$$y_n = \delta_n - 0.5\cdot\delta_{n-1} + a\cdot y_{n-1} + b\cdot y_{n-2}.$$

If $y_n = \cos(\frac{\pi}{3}n)\cdot u_n$, find a and b.

(b) Let $x_n = n\cdot u_n$ and $y_n = (0.5)^n\cdot u_n$. If $z_n = x_n\cdot y_n$, find a recurrence relation similar to the one in (a) for z_n.

2. **[20%] The following problems are of Boolean Algebra.**

(a) Find the minimal sum of products representation for

$$f(v,w,x,y,z) = \sum m(1,2,3,4,8,10,16,18,21,22,23,28,29,30,31)$$

(b) Simply the following expression to the minimal product of sums.

$$(AB+C')()A+C')(A+B'+DE')(B'+C'+DE')$$

(c) Using only NAND and NOR gates to construct the gating network for

$$h(x,y,z)=\overline{(xy\oplus yz)}$$, where $\oplus$ is the exclusive-or operation.

(d) Prove that $x+xy=x$, $x(x+y)=x$ and $x+yz=(x+y)(x+z)$. You **May NOT** use truth tables to make your proof !!

3. **[10%] Use "big-Oh" forms to express your answer. For example, *O(n)*. Show all details.**

(a) For a sorted list of size n, find the computational complexity if a binary search method is used.

(b) Find the computational complexity for the procedure of multiplication of two *n-by-n* matrices.

4. [5%]

(a) The Maclaurin series expansion for e^x is $e^x = \sum_{i=0}^{\infty}\frac{x^i}{n!}$.

Prove that $\cos(x) = \frac{1}{2}(e^{jx}+e^{-jx})$ where $j=\sqrt{-1}$.

(b) Find the convolution of the following two sequences. 1,1,1,1,1,1 and 1,1,1.