

1. Find the associated eigen (characteristic) values and vectors for : (20%)

$$[A] = \begin{bmatrix} 8 & 9 & 9 \\ 3 & 2 & 3 \\ -9 & -9 & -10 \end{bmatrix}$$

2. Find the approximate value of $e^{-0.02}$ (up to six significant digits). (20%)

(Hint: Taylor series: $f(x) = \sum_{m=0}^{\infty} \frac{f^{(m)}(a)}{m!} (x-a)^m$)

3. Find the motion of the mass-spring system corresponding to the following equation and initial conditions : (20%)

$$4y'' + 8y' + 3y = 425 \sin 2t$$

$$y(0) = -16, \quad y'(0) = -26$$

4. Use Laplace transform to solve the following system equations : (20%)

$$\begin{cases} y_1'' = -ky_1 + k(y_2 - y_1) \\ y_2'' = -k(y_2 - y_1) - ky_2 \end{cases}$$

with $y_1(0) = 1, \quad y_2(0) = 1$

$$y_1'(0) = \sqrt{3k}, \quad y_2'(0) = -\sqrt{3k}$$

5. If a cage contains 100 mice, 3 of which are male, what is the probability that the 3 male mice will be included if 10 mice are randomly selected? (20%)