1. (20%)A system is described by $\dot{x} = Ax + Bu$

$$A = \begin{bmatrix} 0 & 2 \\ 0 & 3 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

   (a) Check the system's stability and controllability. (8 points)

   (b) It is desired that the closed-loop eigenvalues be $\lambda_1 = -3$ and $\lambda_2 = -5$. To accomplish this, the state feedback is used, so that the equation becomes to $\dot{x} = (A - BK)x + Bu$. Please find the value of K. (12 points)

2. (15%)Consider the following system

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ -3 & -4 \end{bmatrix} x + \begin{bmatrix} 2 \\ 0 \end{bmatrix} u$$

   where $u(t) = \begin{cases} 0 & t < 0 \\ e^{-t} & t \geq 0 \end{cases}$

   The initial conditions $\begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

   Please find the response of above system.

3. (20%)A mechanical system can be described as a second order differential equation,

   $$M\ddot{y}(t) + B\dot{y}(t) + Ky(t) = u(t)$$

   If we define a state variables as $x(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = \begin{bmatrix} y(t) \\ \dot{y}(t) \end{bmatrix}$, then derive the dynamic

   equation as $\dot{x}(t) = Ax(t) + Bu(t)$
   $$y(t) = Cx(t)$$

   (a) Please find the matrices A, B and C. (10 points)

   (b) Find the transfer function of the system. (10 points)

4. (20%) For each of the following systems, determine whether or not the system is (i) linear (ii) causal (iii) time invariant?

   The input-output relationship is described as following

   (a) $y(t) = \int_0^t u(\tau)d\tau \qquad t \geq 0 \qquad$ (10 points)

   (b) $y(t) = \int_0^{t+2} u(\tau - 2)d\tau \qquad$ (10 points)

   $u(t)$ is the system input and $y(t)$ is the system output.

   (You have to write down your reasons, not just by guess!)

F E14 9-5

5. (25%) Answer the following questions: (5 points each)

(a) A second order system is described as $\dfrac{1}{(s+1)(s+100)}$, this system can be approximated to be first order system. You will choose $\dfrac{1}{(s+1)}$ or $\dfrac{1}{(s+100)}$, why?

(b) For the impulse response function $g(t,\tau)$, please point out its physical meaning.

(c) Most of the physical components have nonlinear characteristic, please take an example and demonstrate its nonlinear input-output relation.

(d) What is PID controller?

(e) Show the difference between DC motor and Stepping motor.