本試題是否可以使用計算機：☑可使用 ，□不可使用 （請命題老師勾選）

Fluid Mechanics

1. Given a two-dimensional velocity field defined by (30%)

$$\vec{V} = x\vec{i} - y\vec{j}$$

(a) Find the streamline passing point (1, 1).
(b) Find the pathline passing point (1, 0) at $t = 0$.
(c) Calculate the circulation along the closed curve with vertices A(0, 0), B(1, 0), C(1, 1), D(0, 1) in a counterclockwise direction.
(d) Calculate the vorticity field corresponding to the above velocity field.
(e) Is the above flow rotational or irrotational? Why?

2. What is a Newtonian fluid? What is the fluid viscosity? What is the unit of viscosity in a SI system? What is a non-newtonian fluid? Is a blood fluid considered to be a Newtonian or non-newtonian fluid? (20%)

3. An incompressible flow is governed by the following equations (50%)

$$\nabla \cdot \vec{V} = 0 \qquad (1)$$

$$\rho \frac{D\vec{V}}{Dt} = -\nabla p + \mu \nabla^2 \vec{V} \qquad (2)$$

Here $\vec{V}$ is the flow velocity, p the pressure, μ the dynamic viscosity, ρ the density, t the time variable, and notation ∇ the gradient operator defined by

$$\nabla = \vec{i}\frac{\partial}{\partial x} + \vec{j}\frac{\partial}{\partial y} + \vec{k}\frac{\partial}{\partial z}.$$

Note that $\nabla^2 = \nabla \cdot \nabla$.

(a) What principles are used to derive equations (1) and (2)?
(b) Explain the physical meaning of each term in equation (2).
(c) Explain the total derivative, $\frac{Df}{Dt}$, of a function $f(t, x(t), y(t), z(t))$ in a physical sense.
(d) Write out an expression for $\frac{D\vec{V}}{Dt}$.
(e) Write out equation (2) in a component form in the Cartesian coordinate system.