

本試題是否可以使用計算機: 可使用, 不可使用 (請命題老師勾選)

1. Suppose that glucose is infused into the bloodstream of a patient at the rate of 3grams per minute, but that the patient's body converts and removes glucose from its blood at a rate proportional to the amount present(with constant of proportionality 0.02). If $Q(t)$ is the amount present at time t and $Q(0) = 120$, (20%)

- (a) write the differential equation for Q ,
(b) solve this differential equation;
(c) determine what happens to Q in the long run.

2. (a). Find the Fourier transforms of the function. (10%)

$$f(x) = \begin{cases} k & \text{if } 0 < x < a \\ 0 & \text{if } x > a \end{cases}$$

(b). Find the Fourier coefficients of the periodic function $f(x)$. (10%)

$$f(x) = \begin{cases} -k & \text{if } -\pi < x < 0 \\ k & \text{if } 0 < x < \pi \end{cases} \quad \text{and} \quad f(x+2\pi) = f(x).$$

3. Find a unit normal vector \mathbf{n} of the cone of revolution $z^2 = 4(x^2 + y^2)$ at the point $P : (1, 0, 2)$. (20%)

4. Find the Taylor series of the following function with center $z_0 = 1$. (20%)

$$f(z) = \frac{2z^2 + 9z + 5}{z^3 + z^2 - 8z - 12}$$

5. Describe the normal distribution, Binomial distribution, Poisson distribution, Linear Regression. (20%)