※ 考生請注意：本試題不可使用計算機。 請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. Let $f(x,y) = e^{xy} \sin(x+y)$.

   (a) In what direction, starting at $(0, \pi/2)$, is f changing the fastest? (10%)

   (b) In what direction, starting at $(0, \pi/2)$, is f changing at 50% of its maximum rate? (10%)

2. In the solution to the problem

   $u_{xx} + u_{yy} = 0$　　for $0 < x < \pi/2, y > 0$

   $u_x(0, y) = 0,$　　$u_x(\pi/2, y) = 0$

   $u(x, 0) = 1 - x,$ $u(x, y)$ is bounded as $y \to \infty$

   find the coefficient on $\cos(6x)e^{-6y}$. (Skip $\lambda < 0$ case) (15%)

3. Evaluate

   $$\int_\gamma \frac{dz}{e^z - 1}$$

   Where γ is the circle $|z| = 9$. (15%)

4. Find the general solution of the following equation: (15%)

   $$y' = \frac{y - x}{y + x}$$

5. Solve the following initial value problem: (15%)

   $y'' + 3.7y' = 0,$　　$y(-2) = 4, y'(-2) = 0$

6. Find the radius of convergence of the following series: (20%)

   $$\sum_{m=0}^{\infty} \frac{(-1)^m}{5^m}(x - 5)^m$$