

※ 考生請注意：本試題不可使用計算機。請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. 20% Let matrix

$$A = \begin{bmatrix} 0 & -1 \\ 4 & 0 \end{bmatrix}$$

Find its eigenvalues and eigenvectors and write the vector

$$u(0) = \begin{bmatrix} 2 \\ 0 \end{bmatrix} \text{ as a combination of those eigenvectors.}$$

(a) Solve the equation $du/dt = Au$ starting with the same vector $u(0)$ at time $t = 0$.

2. 10% Let $f(z) = \frac{1}{(2-z)(z+3)}$

Write the Laurent series expansion of $f(z)$ for $5 < |z - 2| < \infty$ as a power series of $(z - 2)$

3. 20% (a) Let a and b in equation (1) are given constants, what will the equation be transformed to if $z = \ln x$ is substituted in it.

$$x^2 y'' + axy' + by = 0 \quad (x > 0) \quad (1)$$

- (b) Use the result of part (a) to find the general solution of

$$x^2 y'' - 3xy' + 3y = 0$$

4. 15% Solve the following differential equation:

$$(x - 4x^2 y^3)dy + (4x^4 - y)dx = 0$$

5. 15% Solve the following differential equation, using Laplace transformation method. $y(t)$ is a function of t and $u(t)$ is the unit step function.

$$y'' + 2y = u\left(t - \frac{\pi}{\sqrt{2}}\right) - u(t - \sqrt{2}\pi), \quad y(0)=1, \quad y'(0)=0$$

6. 20% Find the general solution of the following equation:

$$y' = \frac{y-x}{y+x}$$