※ 考生請注意：本試題不可使用計算機。　請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. Solve $4y'' + 4(e^x - 1)y' + e^{2x}y = 0$ (15%)

2. Prove the integral as following

$$\int_{-\infty}^{\infty} e^{-ax^2}dx = \sqrt{\frac{\pi}{a}} \text{ , } a > 0 \qquad (15\%)$$

3. *(a) Find the eigenvalues $(\lambda_1 \geq \lambda_2 \geq \lambda_3)$ of the matrix* $A = \begin{bmatrix} 0 & -1 & 0 \\ -1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ (10%)

　　(b) Obtain an orthogonal matrix P which gives $P^T AP = diag(\lambda_1, \lambda_2, \lambda_3)$ (10%)

4. Find the eigenvalues and eigenfunctions of the following problem. (20%)

　　$y'' + \lambda y = 0;\ y'(0) = y'(L) = 0$

5. Determine the inverse Laplace transform of the function $F(s) = \dfrac{1}{s^2 + 16}e^{-2s}$. (15%)

6. Let vector field $\mathbf{F} = 2yz\mathbf{i} - 4xz\mathbf{j} + xy\mathbf{k}$. Evaluate the value of the surface integral $\iint_{\Sigma} \mathbf{F} \cdot \mathbf{n}\, d\sigma$, where

　　Σ is the sphere of radius 5 about $(-1, 3, 1)$. (15%)