系所組別： 電機工程學系丁組，電腦與通信工程研究所甲組

考試科目： 離散數學

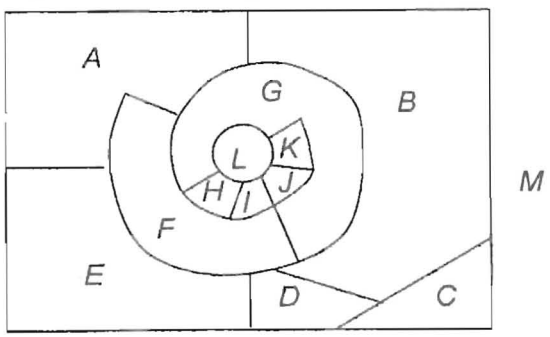
※ 考生請注意：本試題 □可 ☑不可 使用計算機

1. (2 **pts**) (a). How many routes are there from the lower-left corner of an *nxn* square grid to the upper-right corner if we are restricted to traveling only to the right or upward? For example, one such route is shown in a 4x4 grid in the following:

A 4x4 grid with a route from the lower-left Corner to the upper-right corner.

(10 **pts**) (b). How many routes are there from the lower-left corner of an *nxn* square grid to the upper-right corner if we are restricted to traveling only to the right or upward and if we are allowed to touch but not go above a diagonal line from the lower-left corner to the upper-right corner?

2. (5 **pts**) (a). Find the dual of the following map and show that its dual is a planar graph (i.e., the vertices of the dual graph consist of one point in each face of the original graph, including the unbounded face. An edge connects two vertices if the corresponding faces in the original graph are separated by a boundary).



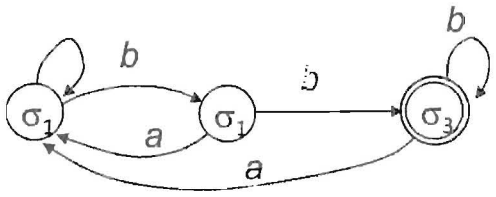
(5 **pts**) (b). Use the result of (a) to show that any coloring of the original map requires at least four colors.

(背面仍有題目,請繼續作答)

※ 考生請注意：本試題 □可 ☑不可 使用計算機

3. (10 pts) Let L be the set of strings accepted by the finite-state automation A of the following graph. Construct a finite-state automaton that accepts the strings

$L = \{x_n \ldots x_1 \mid x_1 \ldots x_n \in L\}$



4. (10 pts) (a). Prove lg $n!$= is Θ (n lg n).
(10 pts) (b). If $f(n)$ is the number of comparisons needed to sort n items in the worst case by a sorting algorithm, then $f(n) = \Omega$ (n lg n).

5. (3 pts) (a). Find the disjunctive normal form of the following function:

x	y	z	f(x, y, z)
1	1	1	1
1	1	0	1
1	0	1	0
1	0	0	1
0	1	1	0
0	1	0	0
0	0	1	1
0	0	0	1

(10 pts) (b). Design the circuit using only NAND gates to compute the above function.

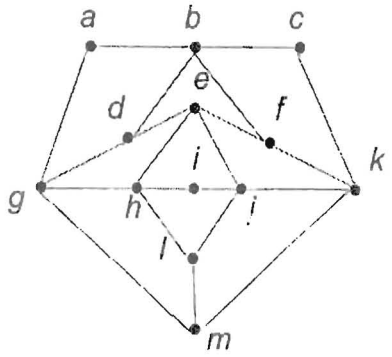
6. (15 pts) Solve the recurrence relation

$$d_n = 4(d_{n-1} - d_{n-2})$$

subject to the initial conditions $d_0 = 1 = d_1$.

※ 考生請注意：本試題 □可 ☑不可 使用計算機

7. (10 **pts**) Determine whether or not the following graph contains a Hamiltonian cycle. If there is a Hamiltonian cycle, exhibit it; otherwise, give an argument that shows there is no Hamiltonian cycle.



8. (5 **pts**) (a). Use mathematical induction to show that

$$f_n^2 = f_{n-1}f_{n+1} + (-1)^{n+1} \qquad \text{for all } n \geq 2$$

where $\{f_n\}$ is Fibonacci sequence (i.e., $f_n = f_{n-1} + f_{n-2}$, for all $n \geq 3$, $f_1 = 1$, $f_2 = 1$).

(5 **pts**) (b). Based on the result of (a), show that for $n \geq 2$

$$f_n = \frac{f_{n-1} + \sqrt{5f_{n-1}^2 + 4(-1)^{n+1}}}{2}$$