※ 考生請注意：本試題不可使用計算機

1. From a faculty of six professors, six associate professors, ten assistant professors, and twelve instructors, a committee of size six is formed randomly. What is the probability that
   (a) (8%) there are exactly two professors on the committee;
   (b) (7%) all committee members are of the same rank?

2. (15%) Let X be a random integer from the set $\{1, 2, \ldots, N\}$. Find $E(X)$, $\mathrm{Var}(X)$, and σ_X.

3. Let X be a geometric random variable with parameter p, and n and m be nonnegative integers.
   (a) (5%) For what values of n is $P(X = n)$ maximum?
   (b) (5%) What is the probability that X is even?
   (c) (10%) Show that the geometric is the only distribution on the positive integers with the memoryless property: $P(X > n + m \mid X > m) = P(X > n)$.

4. (25%) Mark each of the following statements True (T) or False (F).
   (a) If a square matrix A is not invertible, then $A + I$ is invertible, where I is the identity matrix of the same size as A.
   (b) Let W be a subspace of an inner product space V, and $W^{\perp}$ be the orthogonal complement of W. In general, we have $W \cup W^{\perp} = V$.
   (c) We can transform any linear independent set of non-zero vectors into an orthogonal set of vectors by the Gram-Schmidt process.
   (d) If A and B are two $n \times n$ non-invertible matrices, then AB is also non-invertible.
   (e) Let T be a linear transformation from a vector space V to a vector space W. Define a transformation $S : \mathbf{v} \rightarrow T(\mathbf{v}) + \mathbf{w}_o$ from V to W, where $\mathbf{w}_o$ is a constant vector in W. Then S is also a linear transformation from V to W.

5. Suppose that A is a 5×3 real matrix of rank 3. Let $W = A^T A$ and $S = AA^T$.
   (a) (10%) Find the ranks of W and S.
   (b) (5%) Explain why $\lambda = 0$ is an eigenvalue of S.
   (c) (10%) What is the (algebraic) multiplicity of the eigenvalue $\lambda = 0$ of S?