系所組別：電機工程學系丁組、電腦與通信工程研究所甲組

考試科目：離散數學　　考試日期：0222，節次：3

※ 考生請注意：本試題不可使用計算機。　請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. (15%) A single pair of rabbits (male and female) is born at the beginning of a year. Assume the following conditions: First, rabbit pairs are not fertile during their first month of life but thereafter give birth to one new male/female pair at the end of every month. Second, no rabbits die. Third, the initial number of rabbit pairs $F_0 = 1$

a. Let F_k be the number rabbit pairs at the end of month k. Find a recurrence relation for F_k, for each integer k>=1.

b. Use mathematical induction to prove that for all integers k>=0,

$$F_{k+1}^2 - F_k^2 = F_{k-1}F_{k+2}$$

2. (10%) In the computer language Python, identifiers must start with one of 53 symbols: either one of the 52 letters (upper/lower-case alphabet) or an underscore (_). The initial character may stand alone, or it may be followed by any number of additional characters chosen from a set of 63 symbols: the above 53 symbols allowed as an initial character plus the ten digits. There are 31 reserved keywords which has less than or equal to eight character in length, such as if, print, etc, may not be used as identifiers. How many Python identifiers are there that are less than or equal to eight characters in length?

3. (15%) A person giving a party wants to set out 15 assorted cans of soft drinks for his guests. He shops at a store that sells five different types of soft drinks. Please answer the following three questions, you are required to write down the equations and the exact final answers.

a. How many different selections of cans of 15 soft drinks can he make?

b. If beer is one of the types of soft drink, how many different selections include at least six cans of beer?

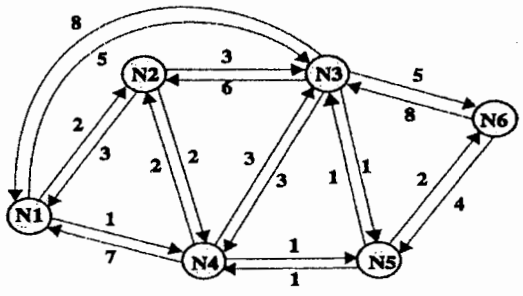
c. If the store has only five cans of beer but at least 15 cans of each other type of soft drink, how many different selections are there?

4. (20%) In a network represented by a graph. Let s be the source node. w(i,j) be the link cost from node i to node j, where w(i,i)=0 and w(i,j)=∞ if the two nodes are not directly connected. L(n) be the cost of the least-cost path from node s to node n that is currently known to the algorithm; at termination, this is the cost of the least-cost path in the graph from s to n. The steps of a least-cost routing algorithm are given below:

1. T={s}, L(n)=w(s,n), for $n \neq s$.
2. Find $x \in T$ such that $L(x)=\min_{j \in T} L(j)$, add x to T and update L(x)
3. $L(n)=\min[L(n), L(x)+w(x,n)]$ for all $n \in T$.

(a) Let s=1. List the values of T, L(2), L(3), L(4), L(5), L(6) in each iteration

(b) Give the name this well-known algorithm.



(背面仍有題目,請繼續作答)

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5. (20%) (a) Design a finite-state automaton A that accepts the sets of all strings of 0's and 1's such that the number of 1's in the string is divisible by 3. (b) Give a regular expression that defines L(A).

6. (20%) What is the purpose of procedure P and what is the output S of the algorithm Select_k?

```
Algorithm Select_k (X, n, k);
Input: X (an array in the range 1 to n), and k (an integer)
Output: S
begin
    if (k<1) or (k>n) then print "error"
    else S:= Select(1,n,k)
end
```

```
procedure Select (Left, Right, k);
begin
    if Left=Right then
        Select := Left
    else
        Middle := P (X, Left, Right)
        if Middle-Left +1 >=k then
            Select(Left,Middle,k)
        else
            Select(Middle+1,Right,k-(Middle-Left+1))
end
```

```
procedure P (X, Left, Right)
Input: X, Left, Right
Output: X and Middle such that X[i]<=X[Middle] for all i<=Middle and X[j]>X[Middle] for all j>Middle
```