國立成功大學104學年度碩士班招生考試試題

系所組別：電機工程學系、電腦與通信工程研究所甲丁組、電機資訊學院-微電奈米聯招

考試科目：工程數學

考試日期：0211，節次：3

※ 考生請注意：本試題不可使用計算機。 請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. (17%) Solve
$$\frac{\partial^2 z}{\partial t^2} = 4\left(\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2}\right)$$
for $0 < x < 2\pi, 0 < y < 2\pi, t > 0$ with following initial and boundary conditions.
$z(x,0,t) = z(x,2\pi,t) = 0 \quad \text{for } 0 < x < 2\pi, t > 0,$
$z(0,y,t) = z(2\pi,y,t) = 0 \quad \text{for } 0 < y < 2\pi, t > 0,$
$z(x,y,0) = 0 \quad \text{for } 0 < x < 2\pi, 0 < y < 2\pi,$
$\frac{\partial z}{\partial t}(x,y,0) = 1 \quad \text{for } 0 < x < 2\pi, 0 < y < 2\pi.$

2. (15%) Consider Bessel function $J_\nu(x)$ of order $\nu \geq 0$ with j_n as its n-th zero. Prove that the functions $\sqrt{x}J_\nu(j_n x)$, for $n = 1, 2, 3, \ldots$, are orthogonal on $[0,1]$ in the sense that
$$\int_0^1 xJ_\nu(j_n x)J_\nu(j_m x)dx = 0 \quad \text{if } n \neq m.$$

3. (15%) Find a general solution of the following differential equation
$y'' - 2y' + y = e^x sinx$

4. (18%) Find the following initial value problem
$$y^{IV} + 3y'' - 4y = 0$$
y(0)=0, y'(0)=-20, y''(0)=0, y'''(0)=80

5. Evaluate the Cauchy principal value of
(a) $\int_{-\infty}^{\infty} \frac{1}{x^4+1} dx$ (10%)
(b) $\int_0^{\infty} \frac{xsinx}{x^2+9} dx$ (10%)

6. Use contour integration to evaluate
$\int_0^{\infty} \frac{(lnx)^2}{x^2+1} dx$ (15%)