國立成功大學104學年度碩士班招生考試試題

系所組別：電腦與通信工程研究所乙組

考試科目：通信數學

考試日期：0211，節次：3

※ 考生請注意：本試題不可使用計算機。 請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. (15%) Suppose that you randomly select a number from an open interval between 0 and 1, i.e., the interval (0,1). Let the number selected be denoted as X.
   (a) Is X a discrete random variable (RV) or a continuous RV? Justify your answer.
   (b) What is the probability that the number selected is 0.5? Justify your answer.
   (c) Use three different ways (in a mathematical language) to describe the *distribution* of X.

2. (10%) Consider a RV X whose expected value and variance exist. You are asked to find a constant c such that it best represents X in the sense that c minimizes the mean square error (MSE), where the MSE is defined as $E[(X-c)^2]$ where $E[\cdot]$ represents the expectation.
   (a) Find c.
   (b) [continued from part (a)] Find the minimum value of the MSE.

3. (10%) The jointly continuous random variables X and Y have a joint probability density function (pdf) that is uniform over the region defined by $\{(x,y)|\,0<x<1, |y|<x\}$.
   (a) Determine the conditional pdf $f_{X|Y}(x|y)$.
   (b) Find the conditional expectation $E[X|Y=y]$ for $|y|<1$.

4. (15%) The RV Z is a Gaussian RV; its mean and variance are denoted as m and σ^2, respectively. Let $X=|Z-m|$. Find the variance of X.

5. (20%) Mark each of the following statements True (T) or False (F). (Need not to give reasons.)
   (a) For a square matrix M, if the columns of M are linearly independent, then the rows of M are also linearly independent.
   (b) For a square matrix M, if the columns of M form an orthonormal set, then the rows of M also form an orthonormal set.
   (c) For an $m \times n$ matrix A, if the columns of A are linearly independent, then AA^T is an invertible matrix.
   (d) If both A and B are $n \times n$ symmetric matrices, then both AB and BA are also symmetric matrices.

6. (10%) Let T be a linear transformation from a vector space V to another vector space W. Suppose that the dimensions of V and W are 4 and 6, respectively. If $\text{rank}(T)=2$, find $\text{nullity}(T)$, which is the dimension of the null space of T.

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7. (20%) Let A and B be two $n \times n$ matrices, and $C = \begin{bmatrix} A & O \\ O & B \end{bmatrix}$, where O is the $n \times n$ zero matrix. Choose the true statement(s) from the following.

   (a) If both A and B are invertible, then C is also invertible.

   (b) If both A and B are diagonalizable, then C is also diagonalizable.

   (c) If both A and B are positive-definite, then C is also positive-definite.

   (d) The rank of C is the sum of ranks of A and B.