國立成功大學104學年度碩士班招生考試試題

系所組別：電腦與通信工程研究所丙組

考試科目：電磁數學

考試日期：0211，節次：3

※ 考生請注意：本試題不可使用計算機。 請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. (15%) Solve the initial-value problem

$$y^{(\mathrm{iv})} - y = 0\ ;\ y(0) = 1,\ y'(0) = y''(0) = y'''(0) = 0$$

for $y(x)$.

2. (15%) Check if integral $I = \int_2^{\infty} \frac{\sin x}{3x^2+1} dx$ converges or not? why? Then, how about $I = \int_3^{\infty} \frac{dx}{x \ln x}$, discuss its convergence.

3. (20%) If $F(s)$ is the Laplace Transform of a periodic function $f(t)$, then find the inverse of $F(s) = \frac{2}{s^2} - \frac{4}{s} \frac{\mathrm{e}^{-2s}}{1-\mathrm{e}^{-2s}}$ and sketch it.

4. (20%) Mark each of the following statements True (T) or False (F). (Need not to give reasons.)

   (a) For a square matrix M, if the columns of M are linearly independent, then the rows of M are also linearly independent.

   (b) For a square matrix M, if the columns of M form an orthogonal set, then the rows of M also form an orthogonal set.

   (c) For an $m \times n$ matrix A, if the columns of A are linearly independent, then $A^T A$ is an invertible matrix.

   (d) If both A and B are $n \times n$ symmetric matrices, then AB is also a symmetric matrix.

5. (10%) Let T be a linear transformation from a vector space V to another vector space W. Suppose that the dimensions of V and W are 5 and 7, respectively. If rank$(T) = 3$, find nullity(T), which denotes the dimension of the null space of T.

6. (20%) Let A and B be two $n \times n$ matrices, and $C = \begin{bmatrix} A & O \\ O & B \end{bmatrix}$, where O is the $n \times n$ zero matrix. Choose the true statement(s) from the following.

   (a) If both A and B are invertible, then C is also invertible.

   (b) If both A and B are diagonalizable, then C is also diagonalizable.

   (c) If both A and B are positive-definite, then C is also positive-definite.

   (d) The rank of C is the sum of ranks of A and B.