國立成功大學 105 學年度碩士班招生考試試題

系　所：電腦與通信工程研究所

考試科目：工程數學

考試日期：0228，節次：3

※ 考生請注意：本試題不可使用計算機。　請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. There are 6 question carrying 100 marks.
2. Credits will be given only when the question numbers and corresponding answers are indicated clearly.

1. (20 points) Let $u(x, y)$ denote the temperature distribution in a homogeneous, thin, flat plate covering the rectangle D as shown in Fig. 1. Solve the following Dirichlet problem

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0 \text{ on } D,$$
$$u(x, 0) = 0 \text{ for } 0 \le x \le a,$$
$$u(0, y) = 0 \text{ for } 0 \le y \le b,$$
$$u(a, y) = 0 \text{ for } 0 \le y \le b,$$
$$y(x, b) = x(x - a) \text{ for } 0 \le x \le a,$$

where $a \ge 0$ and $b \ge 0$.

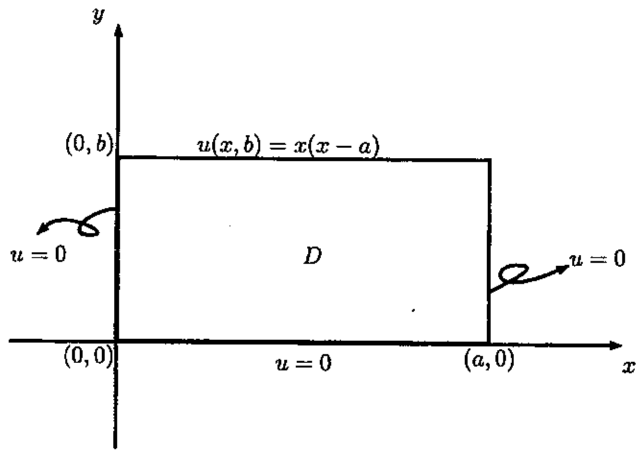

Figure 1: Rectangle D in Problem 1.

2. (30 points) Denote $\mathcal{F}[f(t)](w) = \hat{f}(w)$ the Fourier transform of function $f(t)$. Also, $\mathcal{F}^{-1}[\hat{f}(w)]$ represents the inverse Fourier transform of $\hat{f}(w)$.
   (a) (10 points) $\mathcal{F}\left[H(-ct)e^{act}\right]$, where $H(t)$ is the Heaviside function with $a > 0$ and $c \neq 0$.
   (b) (10 points) Apply Fourier transform to solve the following differential equation.

$$y' + 8y = H(-t)e^{8t}.$$

   (c) (10 points) $\mathcal{F}^{-1}\left[\frac{e^{(\frac{1}{2}w-\frac{1}{2})i}}{1-(1-w)i}\right]$.

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3. (15 points) Solve the following initial value problem. Express the solution in terms of $f(t)$.

$$y^{(4)} - 11y'' + 18y = f(t),$$
$$y(0) = y'(0) = y''(0) = y'''(0) = 0.$$

4. (10 points) Find the phase angle form of the Fourier series of function $f(x)$ in Fig. 2.

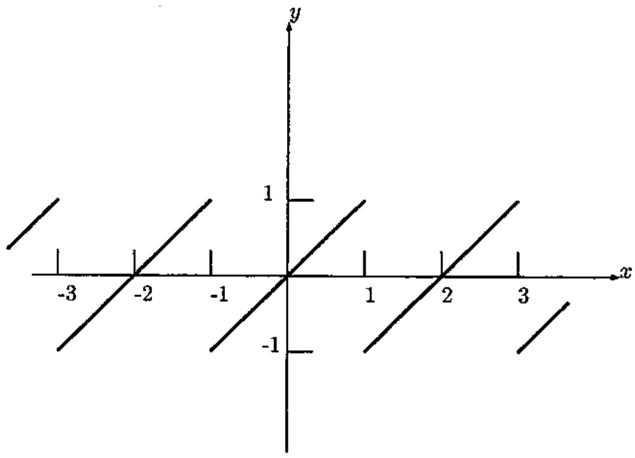

Figure 2: $f(x)$ in Problem 4.

5. (15 points) Consider the use of Frobenius theorem to solve the following different equation.

$$xy'' + (1-x)y' + y = 0.$$

(a) (5 points) Show that $x = 0$ is a regular singular point of the above differential equation.

(b) (5 points) Find all the roots of the indicial equation associated with the above differential equation.

(c) (5 points) Given $y_1(x) = 1-x$ is one of the solutions of the above differential equation, write the second solution which is linearly independent from $y_1(x)$ in the form of a series solution.
[**Notice**] You only need to show the general expression of the series solution without explicitly solving the constant coefficients in the series solution.

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6. (10 points) True or False, and multi-choice questions.
[**Notice**] If you think the answer is "True", answer "T". Otherwise, answer "F". For multi-choice questions, indicate all your choices clearly. No credits will be given if your answers do not follow the required format.

(1) For the Bessel function of the first kind $J_\nu(x)$, if $J_\nu(a) = J_\nu(b) = 0$ and $a \neq b$, then there exists a constant c for $a < c < b$ such that $J_{\nu-1}(c) = 0$.

(2) For the Legendre polynomial $P_5(x)$, the integration $\int_{-1}^{1}(2x)\cdot P_5(x)dx$ is equal to $x^2/2$.

(3) For a Sturm-Liouville problem on the interval $[a, b]$,
(a) there may exist only one eigenvalue;
(b) if λ is an eigenvalue, so is $-\lambda$;
(c) if λ is an eigenvalue, then λ and its conjugate must be identical;
(d) if φ is an eigenfunction, then 2φ is also an eigenfunction;
(e) The Fourier series is a certain kind of eigenfunction expansions

(4) If $f(t) = t^4$ and $g(t) = \sin(t)$, then
(a) the Fourier integral of $f(t)\cdot g(t)$ contains only sine terms;
(b) the Fourier transform of $f(t)$ is twice its Fourier cosine transform;
(c) the Fourier cosine integral of $g(t)$ is exactly the same as its Fourier integral;
(d) the Fourier integral of $|f(t)|$ converges to $f(t)$;
(e) none of the above is correct.

(5) The differential equation $y' + \cot(x)y + |x| = 0$ is not linear because the coefficient function $\cot(x)$ is non-linear.