

※ 考生請注意：本試題不可使用計算機。請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

請按題號作答

1. (15%) Suppose x and y are positive real numbers such that the geometric mean is different from the arithmetic mean. Then $x \neq y$. Please proof by contraposition.

2. (15%) " $\equiv \pmod n$ " is an equivalence relation. Let \mathbf{Z}/n be the set of equivalence classes formed by this equivalence relation. We use $[k]$ to denote the equivalence class. For example, the elements of $\mathbf{Z}/3$ are

$[0]=\{\dots,-6,-3,0,3,6,\dots\}$

$[1]=\{\dots,-5,-2,1,4,7,\dots\}$

$[2]=\{\dots,-4,-1,2,5,8,\dots\}$

Let $[a]$ and $[b]$ be equivalence classes in \mathbf{Z}/n . Prove $x+y \in [a+b]$ and $xy \in [ab]$.

3. (15%) Let X be a set. A list of elements of X is given by the following rules:

1. x where $x \in X$

2. L, x where $x \in X$ and L is a list of elements of X

Let L be a list, where X is the set of real numbers. Define a function $\text{Sum}(L)$ recursively as follow:

1. If $L=x$, a single number, then $\text{Sum}(L)=x$.

2. If L' is a list and $L=L', x$, then $\text{Sum}(L)=\text{Sum}(L')+x$.

Calculate $\text{Sum}(3,1,4,2)$ step by step by using the above definition.

4. (15%) A five-digit number is formed according to the following rules:

The number contains only the digits 1,2,3,4, and these digits may be repeated or unused.

The first digit is 1.

If $D_1 D_2$ are consecutive digits and $D_1 \neq 4$, then $D_1 < D_2$.

If $D_1 D_2$ are consecutive digits and $D_1 = 4$, then $D_2 = 1$.

How many numbers of this type are there? Use a decision tree to arrive at your answer.

5. (10%) Suppose that $g(n) \in O(f(n))$, prove $f(n)+g(n) \in \theta(f(n))$.

6. (10%) Find a theta notation in terms of n for the number of times the statement $x=x+1$ is executed.

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j=n
while (j>=1) {
    for i=1 to j
        x=x+1
    j=j/2
}
    
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7. (10%) Suppose there is a finite set N of nonterminal symbols, and a finite set T of terminal symbols. Let $G(N,T,P,D)$ be a grammar, where $N=\{D\}$, $T=\{d,+,-\}$, $P=\{D \rightarrow D-D++D-D, D \rightarrow d, + \rightarrow +, - \rightarrow -\}$, where P is the set of productions and D is the starting symbol. Assume all symbols must be replaced simultaneously using the above productions. We interpret the symbol d as a command to draw a straight line of a fixed length in the current direction; we interpret $+$ as a command to turn right by 60° ; and $-$ as a command to turn left by 60° . If the first move is horizontal to the right, the string $d-d++d-d \in L(G)$

(a) draw $d-d++d-d$

(b) derive the next longest string in $L(G)$ and draw it.

8. (10%) Design the state diagram for a detector to detect the occurrence of a string "110".