

※ 考生請注意：本試題可使用計算機。請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. (10%) In order to determine the impulse response of an unknown causal, linear time-invariant (LTI) system, Tang feeds the following input  $x[n]$  to the system:

$$x[n] = 0, \text{ if } n < 0; x[n] = 1, \text{ if } n \geq 0.$$

The corresponding output  $y[n]$  is given by the following:  $y[n] = 0$ , if  $n < 0$ ;  $y[n] = 8, 12, 14, 15, 15.5$ , for  $n = 0, 1, 2, 3, 4$ , respectively;  $y[n] = 15.75$ , if  $n \geq 5$ .

- (a) Find the impulse response of this system.  
 (b) Let  $\mathbf{y} = [y[0], \dots, y[5]]^T$  and  $\mathbf{x} = [x[0], \dots, x[5]]^T$ . The input-output relationship of this system can be written as  $\mathbf{y} = \mathbf{H}\mathbf{x}$ . Determine the matrix  $\mathbf{H}$ .
2. (14%) Consider the following two-path channel:

$$y(t) = \frac{1}{\sqrt{2}}s(t) - \frac{1}{\sqrt{2}}s(t - \tau_0)$$

where  $s(t)$  and  $y(t)$  represent the channel input and output, respectively, and  $\tau_0$  is a constant that represents the propagation delay. It can be shown that the channel is an LTI system; the impulse response is denoted as  $h(t)$ .

- (a) Determine  $h(t)$ .  
 (b) For an input signal  $s(t) = \cos(2\pi f_0 t)$  with  $f_0 = \frac{1}{6} \times 10^6$  Hz, determine  $y(t)$ . Assume that the propagation delay is  $\tau_0 = 1 \times 10^{-6}$  sec. (Note: 答案必須是  $A \cos(2\pi ft + \theta)$  的型式。)  
 (c) Suppose that the spectrum of the input  $s(t)$  is centered at  $1.5 \times 10^6$  Hz and has a bandwidth of 100 kHz. If you are asked to design an LTI system (i.e., a linear equalizer) at the receiver that processes  $y(t)$  properly to recover  $s(t)$ . Determine the frequency response of the equalizer. Again, assume that  $\tau_0 = 1 \times 10^{-6}$  sec.
3. (5%) Consider a transmission scheme based on pulse-code modulation (PCM). The message signal is bandlimited by an anti-aliasing filter such that components above 3400 Hz are eliminated. The resulting signal is sampled at a rate of 8 kHz. A uniform quantizer with 256 levels is used to quantize the samples before encoding to binary digits (bits). Each bit is transmitted using a baseband rectangular pulse. Let us assume that the bandwidth occupied by such a pulse of width  $T$  sec is approximately  $\frac{1}{T}$  Hz. Determine the transmission bandwidth of such a scheme.

## 4. (21%) 簡答、填充、選擇題

- (a) [選擇題] Gaussian noise 是指 noise 的何種特性呈高斯分布：(A)取樣值。(B)瞬間功率。(C)功率頻譜密度(power spectral density)。(D)頻譜。(E)振幅頻譜(magnitude spectrum)。
- (b) [選擇題] 訊雜比(SNR, signal-to-noise ratio)為 20 dB 表示：(A)訊號電壓之絕對值是雜訊電壓之絕對值的 20 倍。(B)訊號電壓之絕對值是雜訊電壓之絕對值的 100 倍。(C)訊號功率是雜訊功率的 10 倍。(D)訊號功率是雜訊功率的 100 倍。(E)以上皆非。
- (c) 在 AWGN (additive white Gaussian noise) channel 中，將接收訊號放大  $m$  倍，則 SNR 會變為放大前之 \_\_\_\_ 倍。(填充)
- (d) Is  $x[n] = \cos(0.4n)$  a periodic signal? Justify your answer. (簡答)
- (e) Determine the fundamental period of  $x[n] = \cos\left(\frac{4\pi n}{25}\right)$ . (簡答)
- (f) 解釋為何在 FDM (frequency-division multiplexing) 系統中需使用 guard band。 (簡答)
- (g) 解釋為何 message signal 在調變過程中常被乘上高頻的載波(如  $\cos(2\pi f_c t)$ )? (簡答)

## 5. (22%) Given the following signals:

$$S_A(t) = A \cos\left(\frac{\pi(t - T/2)}{T}\right) \Pi\left(\frac{t - T/2}{T}\right)$$

and

$$S_B(t) = \frac{B}{2} \left\{ 1 + \cos\left(\frac{2\pi(t - T/2)}{T}\right) \right\} \Pi\left(\frac{t - T/2}{T}\right)$$

Assume that they are used in a binary digital data transmission system under the impact of AWGN. The priori-probabilities of  $S_A(t)$  and  $S_B(t)$  are equal. Note that the optimal receiver is the match filter.

- (1) (15%) Derive the error probability  $P_E$ .
- (2) (7%) Write down the optimum detection threshold.
6. (10%) Consider the marker code 10111000. Find the Hamming distance  $h$  between all possible shifts of it and the received sequence 10110 10110 00011 10101. Is there a unique match to within  $h = 1$  and this received sequence? If so, at what delay does it occur?
7. (18%) On the basis of null-to-null bandwidths, given the required transmission bandwidth to achieve a bit rate of 10 Kbps under the AWGN channel using the following modulation scheme:
- (1) (4%) 16-QAM
- (2) (4%) 32-PSK
- (3) (5%) 8-FSK, coherent
- (4) (5%) 16-FSK, noncoherent