國立成功大學105學年度碩士班招生考試試題

系　　所：電腦與通信工程研究所

考試科目：通信數學

考試日期：0228，節次：3

※ 考生請注意：本試題不可使用計算機。　請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. (10%) Let a point be randomly selected from the interior of a triangle; the base of the triangle is of length l, and the height is h from the base. Let X be the distance from the point chosen to the base. Find the probability density function (pdf) of X and plot the pdf.

   Hint: Use the following definition. Let S be a subset of the plane whose area is denoted as $A(S)$ [meaning 'area of S']. A point is said to be randomly selected from S if, for any subset R of S with area $A(R)$ [meaning 'area of R'], the probability that the point selected falls in R is given by $A(R)/A(S)$.

2. (10%) A fair die is rolled N times; the outcome of each roll is recorded. What is the expected value of the sum of these N outcomes.

3. (30%) Let X be a uniformly distributed random variable (RV) over the interval $(0,1)$ [i.e., the open interval between 0 and 1]. Let Y be a uniformly distributed RV over the interval $(0,X)$.
   (a) Find $\mathrm{P}(-2 < Y < 0.5|X = 0.9)$.
   (b) Find $\mathrm{E}[Y|X = x]$ for $0 < x < 1$. Explain why your answer is intuitively correct.
   (c) Find $\mathrm{Var}[Y|X = x]$ for $0 < x < 1$.
   (d) Find the joint pdf of X and Y. How do you verify that your answer is a valid pdf?
   (e) Find the marginal pdf of Y.

4. (20%) Let A be an $n \times n$ matrix.

   (a) Suppose that A is non-invertible. Can you find an $n \times n$ matrix B such that AB is an invertible matrix? (Explain your answer.)

   (b) Suppose that A is invertible. Can you find an $n \times n$ matrix B such that AB is an non-invertible matrix? (Explain your answer.)

5. Let T be a transformation from $M^{3\times3}$ to $\mathbb{R}$, defined by $T(M) = \mathrm{tr}(M)$, the trace of M.

   (a) (10%) Is T a linear transformation?

   (b) (5%) Is T a one-to-one transformation?

   (c) (5%) Is T an onto transformation?

   (d) (10%) Find the rank and nullity of T.