編號： 197 **國立成功大學 106 學年度碩士班招生考試試題**

系　所：電腦與通信工程研究所

考試科目：工程數學　　考試日期：0214，節次：3

※ 考生請注意：本試題不可使用計算機。　請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. There are 5 questions carrying 100 marks.
2. Credits will be given only when the question numbers and corresponding answers are indicated clearly.

1. (20 points) Solve the following initial value problem using Laplace transform. Credits will be given ONLY if the solution is obtained by using Laplace transform.

$$y'' + 4y = 1 + H(t-5), y(0) = 1, y'(0) = 0, \tag{1}$$

where $H(\cdot)$ represents the Heaviside function.

2. (30 points) The Fourier series of a function $f(x)$ defined on $x \in [-L, L]$ is given by

$$f(x) = \frac{1}{2}a_0 + \sum_{n=1} a_n \cos(\frac{n\pi x}{L}) + b_n \sin(\frac{n\pi x}{L}).$$

Answer the following questions.

(a) (5 points) Consider function $f(x) = x^2$ on $x \in [-3, 3]$. This function has the Fourier series on [-3,3] given by

$$3 + \frac{36}{\pi^2}\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2}\cos(n\pi x/3). \tag{2}$$

Under which conditions, we can differentiate the series in Eq. (2) term by term to obtain the Fourier expansion of $f'(x)$ on $[-3, 3]$ (select all answers that are correct)?

(a) f is continuous.

(b) f' is piecewise continuous.

(c) $f(x) \to 0$ as $x \to \infty$.

(d) $f(-3) = f(3)$.

(e) $\int_{-\infty}^{\infty} |f(x)| dx < \infty$.

(b) (10 points) The complex Fourier series of $f(x)$ for $x \in [-3,3]$ is

$$\sum_{n=-\infty}^{\infty} d_n e^{inw_0 x}.$$

where d_n can be obtained as

$$d_n = a \cdot e^{-in\pi} + b \cdot e^{in\pi}. \tag{3}$$

Derive the coefficients a and b in Eq. (3).

(c) (10 points) Find the Fourier cosine integral representation of $g(x)$ defined as

$$g(x) = \begin{cases} x^2, & 0 \le x \le 3 \\ 0, & \text{otherwise.} \end{cases} \tag{4}$$

To get full credits, the Fourier integral cosine coefficient A_w must be arranged in the form of

$$a \cdot \sin(cw) + b\cos(cw).$$

(d) (5 points) State the convergence of the Fourier cosine integral obtained in Question 2(c) for $0 \le x < \infty$.

3. (20 points) Solve the following wave equations.

$$\frac{\partial^2 y}{\partial^2 t} = 4\frac{\partial^2 y}{\partial x^2} + 2xt, \ -\infty < x < \infty, t > 0$$

$$y(x,0) = \sin(x), \frac{\partial y(x,0)}{\partial t} = 0, \ -\infty < x < \infty.$$

4. (15 points) Let $f(z) = \frac{2z}{z^2+1}$.
   (a) (10 points) Find a Laurent series of $f(z)$ in an region $0 < |z - z_0| < R$ about $z_0 = i$ which converges to $f(z)$.
   (b) (5 points) Specify R.

5. (15 points) Use the residue theorem to evaluate the given inverse Laplace transform

$$F(s) = \frac{1}{s^3 + 8}.$$

Express your answer in terms of real-valued functions.