

※ 考生請注意：本試題不可使用計算機。請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

一、複選題：(20 分，每題 5 分，全對才給分)

1. Which of the following statement(s) is (are) true?

- (A) The time complexity of a breath-first algorithm is  $O(V+E)$  which is the fastest search algorithm.
- (B) The time complexity of Dijkstra algorithm is  $O(E \lg V)$  if it is implemented by an array.
- (C) There may exist back edges in a spanning tree generated by a breath-first algorithm.
- (D) The time complexity to delete the maximum value in a heap is  $O(1)$ .

2. Which of the following statement(s) is (are) true (assume each set contains an unique element initially and it is implemented by the disjoint-set forest while the weight rule is applied)?

- (A) In the worst case, find an element in a set of size  $n$  take  $\Theta(\log n)$ .
- (B) The weight rule is to make the taller tree as a subtree of the shorter tree during union.
- (C)  $p$  unions and  $q$  finds can be done in  $O(p+q)$ .
- (D) The kruskal algorithm is implemented by set and the data structure is used to check if the two nodes of a new found edge will form a cycle.

3. Which of the following statement(s) is (are) true?

- (A) Merge sort is faster than the insertion sort algorithm in all conditions.
- (B) The expected time complexity of the quick sort algorithm is  $O(n \log n)$  when using RANDOMIZED-PARTITION.
- (C) The complexity of a comparison based algorithm cannot be faster than  $O(n \log n)$ .
- (D) A LSD Radix sort is not a stable sort.

4. Which of the following statement(s) is (are) true?

- (A) If  $n_1$  and  $n_2$  are the left child and right child of a node  $n$  in a max heap, the value of  $n_1$  cannot be larger than  $n_2$ .
- (B) It takes  $O(\log n)$  to search a maximum value in a binary search tree if the tree has  $n$  nodes.
- (C) A binary search tree is usually implemented by a complete binary tree.
- (D) The time complexity to perform Max Heapify can be as fast as  $O(n)$ .

二、簡答題: (60 分)

1. (20 pts) Give a table which characterizes the priority of the operators.

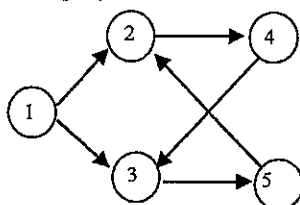
priority	ISP	ICP
(		0
!	1	1
* / %	2	2
+ -	3	3
=	4	4
)	5	

The expression is  $(A + B) * (C - !D / E) \% F$

- (5 pts) Show the equivalent postfix expression if we use a stack to evaluate the postfix expression in C++ from left to right.
- (5 pts) Based on a, please show the status of the stack in each time before you pop data, how many pop operations you have to perform.
- (7 pts) Draw a binary tree representation which corresponds to the sequence of its infix expression.
- (3 pts) Please show the results of the expression  $5 * 2 + 30 - 20 - 80 / 5$  if we reverse the priority of the operators  $* /$  with the priority of the operators  $+ -$  and the evaluation of operators of the same priority will proceed from right to left.

2. (15 pts) Transitive closure graph.

According to the graph G shown in the following:



- (1 pts) Use an adjacency matrix to represent the graph.
- (6 pts) Find the transitive closure matrix  $A^+$  and the reflexive transitive closure matrix  $A^*$  of G.
- (2 pts) Explain the meaning of  $A^+$ .
- (2 pts) Explain the meaning of  $A^*$ .
- (4 pts) If we use a matrix  $A^0$  to represent the distances between any two vertices, where  $t A^0(i, j)$  denotes the shortest distance between two vertices  $i$  and  $j$ , Floyd-Warshall algorithm can find the shortest path between any two vertices by compute a new matrix  $A^k$  from  $A^{k-1}$ . Please list the general recursive equation and explain the equation shortly (hint: what does  $k$  represent in the equation?).

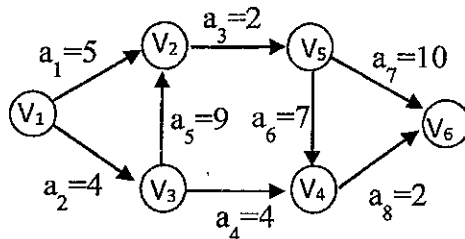
3. (25 pts) Active On Edge (AOV) network.

- a. (2 pts) Please show the adjacency list of the figure (assume the sequence of nodes in a list are arranged according to their numbers in increasing order).
- b. (2 pts) Find a topological sequence of nodes according to the data structure obtained in a.
- c. (6 pts) Find the earliest even occurrence time  $ee(i)$  and the latest even occurrence time  $le(i)$  for each even  $i$  and show the results in a table according to the sequence of the index of even  $i$ .
- d. (8 pts) Find the earliest time,  $e(i)$ , and latest time,  $l(i)$ , for each activity and shows the result in a table according to the sequence of the index of activity  $i$ .
- e. (1 pts) What is the earliest time when the project can be finished?
- f. (2 pts) Please show the slack of each activity.
- g. (2 pts) Which activities are critical and why they are critical?
- h. (2 pts) Please show the activities which can result in reduction of the project length if they speed up.

$$e(i) = ee[k] = \max_{\langle j, k \rangle \in E} \{ee[j] + \text{duration\_on\_}\langle j, k \rangle\}$$

$$l(i) = le[l] - \text{duration\_of\_}a_i$$

$$= \min_{\langle l, j \rangle \in E} \{le[j] - \text{duration\_on\_}\langle l, j \rangle\} - \text{duration\_of\_}a_i$$



三、填充題 (20 分，每題 10 分)

1. Give the following algorithm whose input is a graph G, an edge weight w, and a vertex r in G.

```

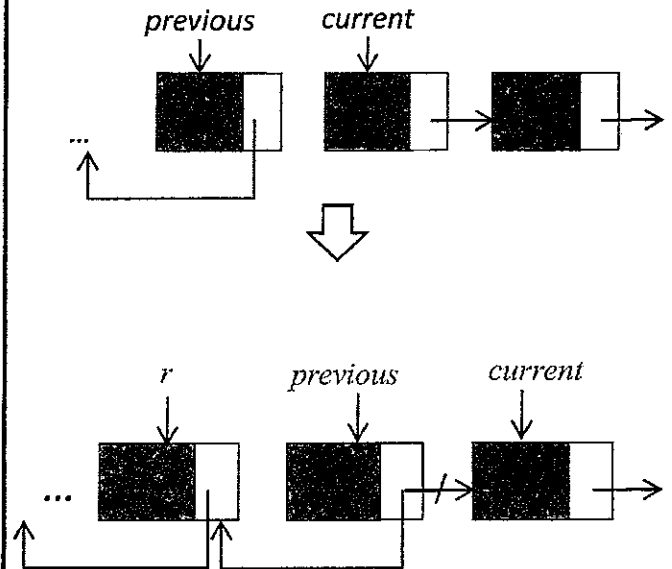
WeightGraph(G, w, r)
for each u ∈ V(G)
    key[u] = ∞
Insert all nodes into Q; // Q denotes a priority queue.
key[r] = 0;
while (Q is not empty)
    u = ExtractMin(Q)
    for each neighbor v of u and v ∈ Q
        if ( _____ A _____ )
            _____ B _____
    
```

- a. (4 pts) Please list the answers of A and B if the algorithm is to find the minimum spanning tree.
- b. (4 pts) Please list the answers of A and B if the algorithm is to find the single source shortest tree.
- c. (1 pts) Please give the time complexity in term of V and E.
- d. (1 pts) Please give the time complexity if we replace Q by an array A.

2. (10 pts) Please reference the figure in the bellow to complete the pseudo code of the reverse which reverses the sequence of nodes in a list.

```

class Chain; // forward declaration
class ChainNode {
friend class Chain;
public:
    ChainNode (int element = 0, ChainNode* next = 0)
        // 0 is the default value for element and next
        {data = element; link = next;}
private:
    int data;
    ChainNode *link;
};
class Chain { // true definition
    ChainNode *first; // has a ChainNode pointer
public:
    void Reverse(); // reverse the sequence of nodes
                    // in a list
};
    
```



```

template <class T>
void Chain<T>::Reverse()
{ // A chain is reversed so that (a1, ..., an) becomes (an, ..., a1)
    ChainNode <T> *current = first,
                *previous = 0; // previous trails current
    while ( ___ A ___ ) {
        ChainNode <T> * ___ B ___;
        ___ C ___;
        ___ D ___;
        ___ E ___;
    }
    first = previous;
}
    
```