

※ 考生請注意：本試題可使用計算機。請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. (20%) Consider the amplitude modulation (AM) signal: $x_c(t) = [m(t) + K]A_c \cos(2\pi f_c t + \theta)$ where $m(t)$ is the real-valued message signal that satisfies $\int_{-\infty}^{\infty} m(t)dt = 0$ (i.e., $m(t)$ does not have the DC component), $K > 0$ is the DC bias, $A_c > 0$ is a constant, $f_c > 0$ is the carrier frequency in Hertz (Hz), θ is a constant phase, and $x_c(t)$ is the transmitted AM signal. Let $x_r(t)$ denote the received signal where we assume that $x_r(t) = x_c(t)$. Assume that the message bandwidth is W Hz.
- (a) (5%) Draw the block diagram of the coherent demodulator whose output would be $m(t)$. You can use an ideal lowpass filter, but you need to specify the bandwidth of it (for example, how the filter bandwidth relates to the message bandwidth). You can assume that the receiver has somehow achieved perfect carrier synchronization prior to the demodulation starts.
- (b) (5%) From a time-domain perspective, show that the coherent demodulator you drew for part (a) does indeed successfully recover the message signal.
Hint: 寫出 demodulator 中各個訊號點的 time-domain expression.
- (c) (10%) From a frequency-domain perspective, show that the coherent demodulator you drew for part (a) does indeed successfully recover the message signal.
Hint: 寫出 demodulator 中各個訊號點的 frequency-domain expression in terms of continuous-time Fourier transform. Please note: 本題並無假設 $m(t)$ 的頻譜是呈某特定形狀，故請勿使用圖解畫頻譜之方式作答。
- [Note]** 上述各小題若僅以 $\theta=0$ 的 special case 作答，將只得到部份分數。
2. (20%) Consider a zero-mean, stationary Gaussian process $X(t)$ whose auto-correlation function is given by $R_X(\tau) = \alpha \text{sinc}(B\tau)$ where $\alpha > 0$, $B > 0$, and $\text{sinc}(z) \equiv \frac{\sin(\pi z)}{\pi z}$.
- (a) (10%) Suppose that you observe $X(t)$ at the time instant $t=t_0$. Determine the probability density function (pdf) of the observed value $X(t_0)$.
- (b) (10%) Suppose that you observe $X(t)$ at the time instants $t=t_0$ and $t=t_0+(2/B)$. Determine the joint pdf of $X(t_0)$ and $X(t_0 + \frac{2}{B})$.
3. (10%) Consider a discrete-time linear time-invariant system whose unit impulse response is denoted as $h(n)$. Let the input of this system be $x[n] = e^{j\omega_0 n}$ where $j = \sqrt{-1}$ and ω_0 is the angular frequency. Find the output of this system, $y[n]$. You need to express $y[n]$ in a product form, i.e., writing $y[n]$ as the product of the input $x[n]$ and a scalar. You need to define this scalar, and specify the relationship of this scalar and $h(n)$. Finally, comment on the importance of this product form of $y[n]$.

4. (28%) Consider the binary digital data communication system illustrated in Fig. 1, in which the transmitted signal consists of a sequence of constant-amplitude pulses of either A or $-A$ units in amplitude and T seconds in duration; it experiences the additive white Gaussian noise with double-sided power spectrum density $N_0/2$ W/Hz to the signal. With the priori probability $P(A) = P(-A) = 1/2$, using the receiver shown in Fig. 2 can lead to the error probability of

$$P_E = Q\left(\sqrt{\frac{2A^2T}{N_0}}\right).$$

Now, please answer the following questions; and You need to show all the derivations in detail.

- (a) (14%) If the timing is off at the receiver so that the integration starts ΔT late (positively) or early (negatively), what will the error probability be? Assume that the timing error is less than one signaling interval.
Hint: Assume a zero threshold and considering two successive intervals, i.e. (A, A) , $(A, -A)$, $(-A, A)$ and $(-A, -A)$.
- (b) (14%) If the threshold “0” is replaced by a non-zero constant “ ϵ ”, what will the error probability be?

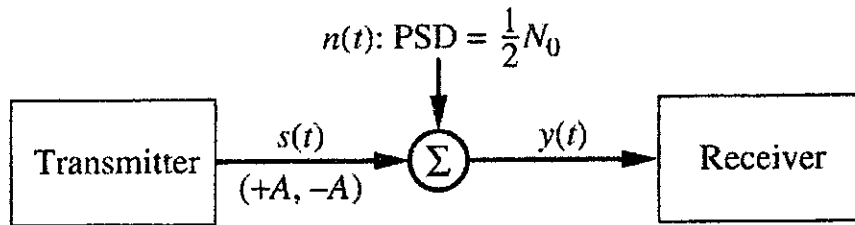


Fig. 1

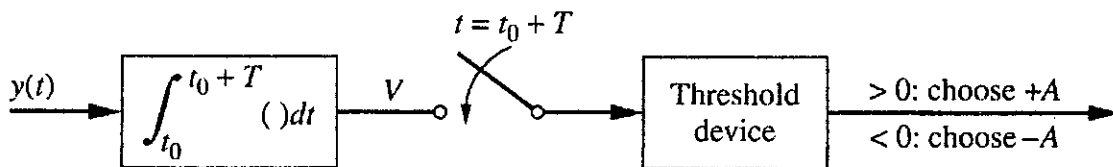


Fig. 2

5. (22%) A serial bit stream, proceeding at a rate of 10 kbps from a source, is given as 101110 000111 010011 (spacing for clarity)
Number the bits from left to right starting with 1 and going through 18 for the right most bit. Associate the odd-indexed bits with $d_1(t)$ and the even-indexed bits with $d_2(t)$ in Fig. 3.
- (a) (4%) What is the symbol rate for $d_1(t)$ and $d_2(t)$?
- (b) (8%) What are the successive values of $\theta_i = \tan^{-1}(d_2(t)/d_1(t))$ assuming QPSK modulation? At what time

intervals may θ_i switch?

(c) (10%) What are the successive values of $\theta_i = \tan^{-1}(d_2(t)/d_1(t))$ assuming OQPSK modulation? At what time intervals may θ_i switch?

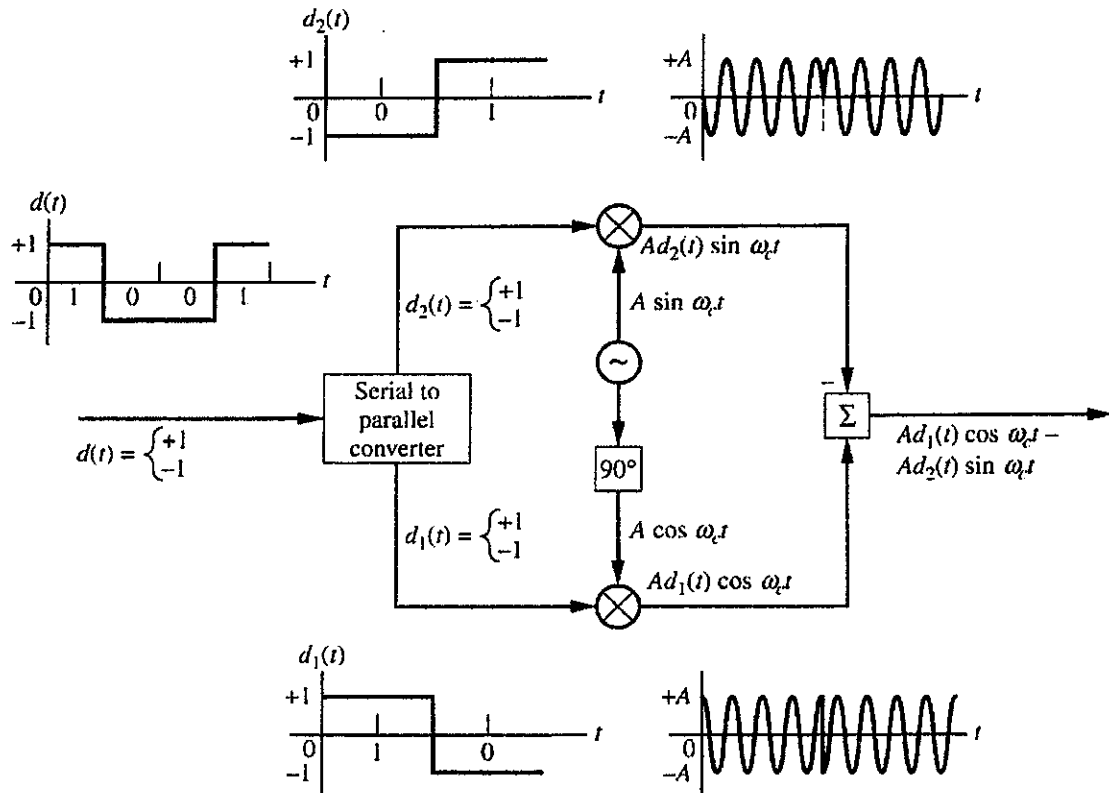


Fig. 3