※ 考生請注意：本試題不可使用計算機。　請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. 10% (a) Find the eigenvalues of

$$C = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \qquad \text{and } C^2 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

10% (b) Find the determinants of C + I and C + 2I

$$\text{Where } I = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

2. 10% (a) Find the Fourier sine series expansion in $[0, \pi]$ of the function h(x) defined in $[0, \pi]$ by h(x)= -1 in $[0, \pi/2]$ and h(x)= 1 in $[\pi/2, \pi]$. Sketch the function represented by the sine series in the symmetric interval $[-\pi, \pi]$.

   10% (b) Find the Fourier cosine series expansion in $[0, \pi]$ of the function h(x) defined in part (a). Sketch the function represented by the cosine series in the symmetric interval $[-\pi, \pi]$.

3. 10% Determine the constant A so that the following function is analytic everywhere.

$$f(z) = \begin{cases} A \frac{\cosh z - 1}{z^2} & if \ z \neq 0 \\ 1 & if \ z = 0 \end{cases}$$

4. 10% 解下列微分方程式:

$$(2xy+3y)dx+(4y^3+x^2+3x+4)dy=0 \ , \ y(0)=1$$

5. 16% 解方程式:

$$y''+9y = x^2\sin 3x$$

6. 24% 矩陣 $A = \begin{bmatrix} 5 & -4 & 2 \\ 3 & -2 & 2 \\ 2 & -2 & 3 \end{bmatrix}$ 求矩陣 A 的特徵值 (5%)、特徵向量 (9%)

   及 e^{At} (10%)