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國立成功大學 107 學年度碩士班招生考試試題

系　所：電腦與通信工程研究所

考試科目：通信數學

考試日期：0205，節次：3

※ 考生請注意：本試題不可使用計算機。　請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. (35%) We start with a stick of length L. We break it at a point that is chosen according to a uniform distribution, and keep the piece that contains the left end of the stick. The length of the piece that is kept is defined as Y. We then repeat the same process on the piece that we were left with, and let X be the length of the remaining piece after breaking for the second time.
   (a) Find the joint probability density function (pdf) of X and Y.
   (b) Find the marginal pdf of X.
   (c) Use the pdf of X to evaluate $E[X]$, the expectation of X.
   (d) Evaluate $E[X]$, by exploiting the relation $X = Y \cdot (X/Y)$.

2. (15%) Consider a test of a certain disease. Let us refer to the disease as disease S.
   (a) If a person takes the test twice and both results are positive, what is the probability that he has disease S? You are given the following information.
   (i) The probability that a random person (drawn from the country of this person) has disease S is y.
   (ii) When a person takes the test twice, the first test has a correct rate of x. That is, if a person has disease S, then the probability that the first test result is positive is x; if the person does not have disease S, then the probability that the first test result is negative is also x.
   (iii) For the second test, the conditional probability that the second test result is the same as the first one is q, no matter the person has disease S or not. That is,
   P(2nd result is positive|1st result is positive, the person has disease S) = q,
   P(2nd result is negative|1st result is negative, the person has disease S)=q,
   P(2nd result is positive|1st result is positive, the person does not have disease S) = q,
   and P(2nd result is negative|1st result is negative, the person does not have disease S)=q.

   (b) [continued from part (a)] Now redo the problem with somewhat different assumptions. All the assumptions in part (a) apply, except assumption (iii). Instead, the outcomes of the two tests are independent, and the correct rate of the second test is the same as the first one. If a person takes the test twice and both results are positive, what is the probability that he has disease S?

3. (10%) Suppose that both A and B are 3×3 matrices. The eigenvalues of A are 1, 2, and 3; while the eigenvalues of B are -1, -3, and 4. Find the determinants of AB and $A(B+I)$, respectively.

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4. (20%) Choose the true statement(s) from the following. (Need not to give reasons.)
   (a) Let V be a vector space, and S_1 and S_2 be two subspaces of V. It is possible that S_1 and S_2 are disjoint, $S_1 \cap S_2 = \phi$, where ϕ denotes the empty set.
   (b) Suppose that A and B are two $n \times n$ matrices. If A has an eigenvalue λ_a and B has an eigenvalue λ_b , then $(A + B)$ has an eigenvalue $(\lambda_a + \lambda_b)$.
   (c) Let T be a linear transformation (operator) on a vector space V. Then T^2 is also a linear transformation (operator) on V.
   (d) For any matrix A, we have $\text{rank}(A^T A) = \text{rank}(AA^T)$

5. Suppose that A and B are two $m \times n$ matrices and $m < n$.
   (a) (10%) Is it possible that AB^T is an invertible matrix? (Give your reason.)
   (b) (10%) Is it possible that $A^T B$ is an invertible matrix? (Give your reason.)
   (We use M^T to denote the transpose of a matrix M.)