

※ 考生請注意：本試題不可使用計算機。請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. 10% (a) Find real a, b , such that $b > 0$ and $a + bi$ is a cube root of -1 .

(b) Write $f(t) = 2 \cos(4t) - 2 \sin(4t)$ in the form $A \cos(\omega t - \phi)$.

2. 20% $M = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \\ 2 & -1 & -1 \end{pmatrix} \quad M^{-1} = \frac{1}{12} \begin{pmatrix} 1 & 1 & 4 \\ a & 7 & -8 \\ b & -5 & 4 \end{pmatrix}$

(a) Find the numbers a and b in the formula for the matrix M^{-1}

(b) Find the solution $\vec{r} \Rightarrow \langle x, y, z \rangle$ to $\begin{cases} x + 2y + 3z = 0 \\ 3x + 2y + z = t \\ 2x - y - z = 3 \end{cases}$ as a function of t

3. 20% Use series to find the general solution of $(1-x^2)y'' - xy' + y = 0$,

where $|x| < 1$, in the form

$$y = \sum_{n=0}^{\infty} a_n x^n$$

with a_0 and a_1 arbitrary.

4. 20% Solve the following differential equation and provide general solution of $y(x)$

(a) $\frac{dy}{dx} + 3x^2y = x^5$ (10%)

(b) $(y - x^2y) \frac{dy}{dx} = x + 1$ (10%)

5. 15% Find the solution of the ordinary differential equation and what are the eigenvalues of the differential equation?

$$y'' + 5y' + 4y = 3 - 2x, \quad y(0)=11/8, y'(0)=1/2$$

6. 15% Find the inverse Laplace transform of

$$F(s) = \frac{s+1}{s^4+2s^3-2s-1}$$