

※ 考生請注意：本試題不可使用計算機。請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

一、複選題：（30 分，每題 10 分，全對才給分）

1. Which of the following statements is(are) true?

- (A) The best case time complexity of Bubble Sort is $\Omega(n)$.
- (B) Quick Sort sorts in place.
- (C) The worst case time complexity of Quick Sort is $O(n^2)$.
- (D) To HEAPIFY a max-heap runs in $O(\lg n)$ time.
- (E) Heap Sort cannot sort in place.

2. Which of the following statements is(are) true?

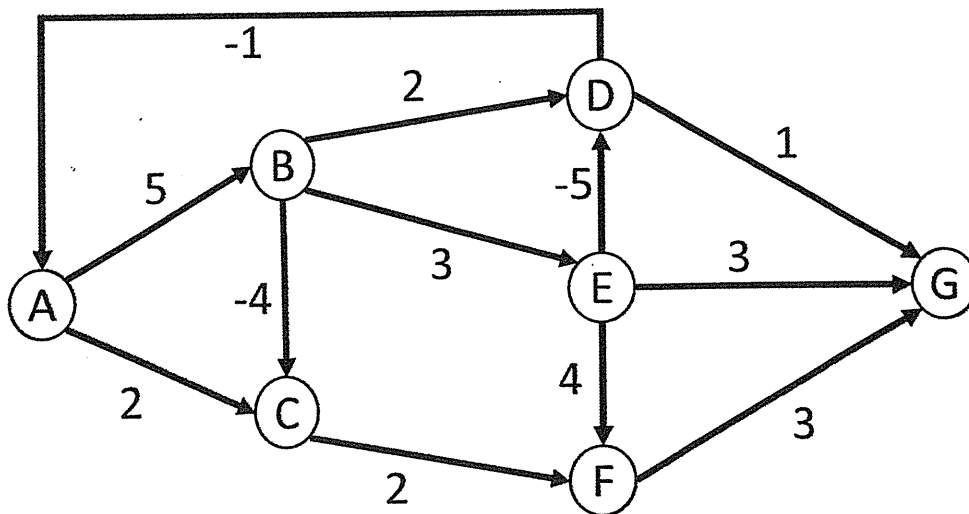
- (A) A binary tree, which can be empty, is always a tree.
- (B) The degree, i.e., number of subtrees, of any node in a binary tree must not exceed two.
- (C) The minimum number of levels of a binary tree with 8 nodes is 4.
- (D) The maximum number of levels of a binary tree with 8 nodes is 8.
- (E) For any nonempty binary tree, if NL is the number of leaf nodes and NT is the number of nodes with degree 2, then $NL=NT+2$.

3. Given an undirected graph $G=(V, E)$, where V is a finite, nonempty set of vertices and E is a set of pairs of vertices called edges, with no self-edge and multi-occurrence of the same edge, which of the following statements is(are) true?

- (A) The maximum number of distinct edges in E in a graph with n vertices is $n(n-1)$.
- (B) If G is a connected graph with n vertices, G must have at least n edges.
- (C) The number of nodes in a spanning tree of G with n vertices is n-1.
- (D) The maximum clique of a complete graph with n vertices has n nodes.
- (E) A complete graph must be connected.

二、問答題：

1. What are the best-case time complexity using “ Θ ” notation to move n ($n > 0$) plates from one of the three Hanoi towers to another? (5 points)
2. What are the contents from Front to Rear in a stack after the following operations, [push(A), push(B), pop(), push(C), pop(), pop(), push(D), push(E), pop(), push(F)]? (5 points)
3. What is the maximum number of nodes in a full K-ray tree (degree of nodes in the tree is no more than K) with height L? (5 points)
4. (A) Please transform the following infix expression “ $A * B - C + (D + E) * F + G / H$ ” to the corresponding postfix expression using the Stack or Queue data structures. Please describe your answer in details. (10 points)
 (B) Please evaluate the result from the above postfix expression using the Stack or Queue data structures if [A=8, B=7, C=6, D=5, E=4, F=3, G=2, H=1]. Please describe your answer in details. (5 points)
5. You are given an empty hash table of size 7 and bucket capacity 1 that uses open addressing. The following sequence of keys is to be inserted: (15, 17, 8, 23, 3, 5). Insert these keys using each of the following hash functions and approaches. Show the resulting hash table step by step. If overflow occurs, say so, indicate the element that causes the overflow. (10 points)
 (A) $h(x) = x \% 7$; linear probing.
 (B) $h(x) = x \% 7$; double hashing with $h_2(x) = x / 7 + 1$.
6. Please compute the all-pair shortest distances in the following graph. Please show your answer step by step. (15 points)



7. The distance between two vectors $x=[x_1, x_2, \dots, x_N]$ and $y=[y_1, y_2, \dots, y_M]$ can be computed by the dynamic time warping (DTW for short). DTW is to find the best mapping path with the minimum distance.

An (N, M) -warping path is a sequence $p = (p_1, \dots, p_L)$ with $p_k = (n_k, m_k) \in [1 : N] \times [1 : M]$ for $k \in [1 : L]$ satisfying the following three conditions.

(i) Boundary condition: $p_1 = (1, 1)$ and $p_L = (N, M)$.

(ii) Monotonicity condition: $n_1 \leq n_2 \leq \dots \leq n_L$ and $m_1 \leq m_2 \leq \dots \leq m_L$.

(iii) Step size condition: $p_{k+1} - p_k \in \{(1, 0), (0, 1), (1, 1)\}$ for $k \in [1 : L - 1]$.

The total cost $c_p(X, Y)$ of a warping path p between X and Y with respect to the local cost measure c is defined as:

$$c_p(X, Y) := \sum_{k=1}^L c(x_{n_k}, y_{m_k})$$

An optimal warping path between X and Y is a warping path p^* having minimal total cost among all possible warping paths. The DTW distance $DTW(X, Y)$ between X and Y is then defined as the total cost of p^* :

$$DTW(X, Y) := c_{p^*}(X, Y) = \min\{c_p(X, Y) \mid p \text{ is an } (N, M)\text{-warping path}\}$$

Please compute the DTW distance of $x=[1, 2, 4, 1]$ and $y=[1, 5, 4, 2]$ using dynamic programming techniques with cost measure c defined as:

$$c(x_i, y_j) = \sqrt{(x_i - y_j)^2}$$

Please show your answer step by step with descriptions. (15 points)