

※ 考生請注意：本試題可使用計算機。請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

For your reference:  $\epsilon_0 = 10^{-9}/36\pi$  (F/m);  $\mu_0 = 4\pi \times 10^{-7}$  (H/m);  $\eta_0 = 120\pi$  ( $\Omega$ )  
 Permittivity  $\epsilon (= \epsilon_r \epsilon_0)$ ; Permeability  $\mu (= \mu_r \mu_0)$ ; Conductivity  $\sigma$

1. Please calculate

- (a) The volume in cylindrical coordinates between  $r = 2$  m and  $r = 3$  m contains a uniform charge density  $\rho$  ( $C/m^3$ ), as shown in Fig. A. Use Gauss's law to find the electric flux density  $\vec{D}$  in all regions. [6%]  
 (b) A spherical conducting shell of radius  $a$ , centered at the origin, has a potential field

$$V(r) = \begin{cases} V_0, & r \leq a \\ V_0 \cdot \frac{a}{r}, & r > a \end{cases}$$

with the zero reference at infinity. Find the electric field  $\vec{E}$  anywhere and the stored electrostatic energy  $W_e$  in this system. [9%]

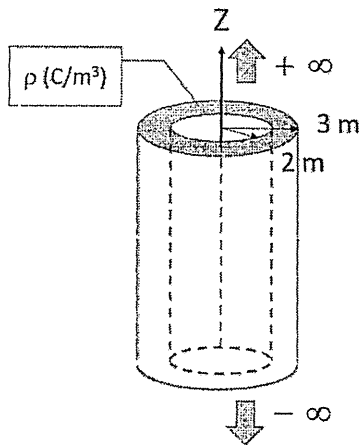


Fig. A

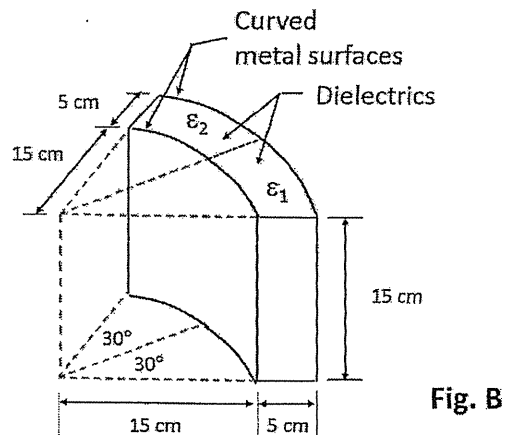


Fig. B

2. Find the total capacitance between the inner ( $r_i = 15$  cm) and outer ( $r_o = 20$  cm) curved metal surfaces as referred to the cylindrical coordinates and shown in Fig. B. The dielectric constants are  $\epsilon_1 = 2.5\epsilon_0$  and  $\epsilon_2 = 4\epsilon_0$ , respectively. Neglect the fringing field effect. [10%]  
 3. In free space, an electromagnetic plane wave propagates along the z-axis. The magnetic field is

$$\vec{H}(z,t) = \frac{0.4}{3} \cos(4 \times 10^7 t - \beta z) \hat{a}_y \quad (\text{A/m}).$$

Find the electric field  $\vec{E}(z,t)$ , the phase constant  $\beta$ , and the wave length  $\lambda$ . [15%]

4. As shown in Fig. C, a rectangular conducting loop with resistance  $R = 0.2 \Omega$  rotates with 300 rpm. The vertical conductor at  $r_1 = 0.03 \text{ m}$  is in a flux density  $\vec{B}_1 = 0.25 \hat{a}_r \text{ (T)}$ , and the conductor at  $r_2 = 0.05 \text{ m}$  is in  $\vec{B}_2 = 0.8 \hat{a}_r \text{ (T)}$ . The vertical conductor length is 0.5 m. Find the induced current in the loop. [15%]

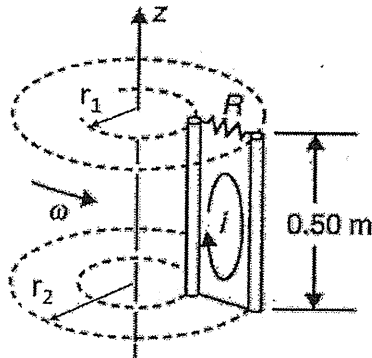


Fig. C

magnetic

5. A parallel-wire transmission line is constructed of #6 AWG copper wire (diameter = 0.162 in;  $\sigma_c = 5.8 \times 10^7 \text{ S/m}$ ) with a 12-inch separation in air. Neglect the internal inductance,
- (a) find the transmission line parameters of  $L$ ,  $C$ ,  $G$ , the dc resistance  $R_{dc}$ , and the ac resistance  $R_{ac}$  at 1 MHz; [10%] and
- (b) find the characteristic impedance, propagation constant, propagation velocity, and wavelength while the operation frequency is at 5 kHz. [10%]
6. A rectangular waveguide having dimensions of  $a = 3.484 \text{ cm}$  and  $b = 1.58 \text{ cm}$  is filled with a dielectric material ( $\epsilon = 2.25 \epsilon_0$ ;  $\mu = \mu_0$ ). The operation frequency is 4 GHz.
- (a) Find the phase constant  $\beta$ , the guided wavelength  $\lambda_g$ , the phase velocity  $v_p$ , the group velocity  $v_g$ , and the wave impedance  $Z$  of the dominant mode. [10%]
- (b) Compute the propagation constant when the operation frequency is 2 GHz. How much does the wave attenuate over the distance of 1 cm? [5%]
7. The radiation fields of a half-wave dipole antenna in phasor expression are given by

$$\vec{E}_\theta = j\eta_0 2I_0 \frac{e^{-jk_r r} \cos\left(\frac{\pi}{2} \cos\theta\right)}{4\pi r \sin\theta} \hat{a}_\theta \quad \text{and} \quad \vec{H}_\phi = \frac{\vec{E}_\theta}{\eta_0} \hat{a}_\phi$$

where  $I_0$  is the current amplitude at the input port of the antenna. Calculate the radiation resistance and directivity of a half-wave dipole antenna. (Note: calculation is necessary.) [10%]