編號： 192

國立成功大學 109 學年度碩士班招生考試試題

系　　所：電腦與通信工程研究所

考試科目：電磁數學

考試日期：0211，節次：3

※ 考生請注意：本試題不可使用計算機。　請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. (25%) Obtain the general solution of the differential equation $x^2y'' + 2xy' + (x^2 - 1)y = 0$.

2. (25%) Find the eigenvalues and corresponding eigenfunctions of the Sturm-Liouville problem.

$$y'' + \lambda y = 0 \quad (0 < x < L)$$

$$y(0) = 0,\ hy(L) + y'(L) = 0 \quad (h > 0)$$

3. (20%) Choose the true statement(s) from the following.

   (a) If M is an invertible matrix, then $M+I$ is also an invertible matrix. (I denotes the identity matrix of the size as M).

   (b) For an $n \times n$ matrix A, if $A^2 = O$, where O denotes the zero matrix, then we have $A = O$.

   (c) For an $n \times n$ matrix M, we have $\text{rank}(M^2) \leq \text{rank}(M)$.

   (d) A real-valued square matrix may have complex eigenvalues and complex eigenvectors.

4. Let A be an $n \times n$ real-valued symmetric matrix, $A^T = A$, and I is an identity matrix of size n.

   (a) (20%) Show that $I + A^2$ is always an invertible matrix.

   (b) (10%) Define a transformation from the space of $n \times n$ real-valued matrices to the space of real numbers as $T(A) = \det(A)$, where $\det(A)$ is the determinant of A. Is T a linear transformation?