

※ 考生請注意：本試題可使用計算機。請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

For your reference: $\epsilon_0 = 10^{-9}/36\pi$ (F/m); $\mu_0 = 4\pi \times 10^{-7}$ (H/m); $\eta_0 = 120\pi$ (Ω)
 Permittivity $\epsilon (= \epsilon_r \epsilon_0)$; Permeability $\mu (= \mu_r \mu_0)$; Conductivity σ

1. A simple capacitive fuel gauge can be built by a cylindrical capacitor, as shown in Fig. A. The diameter of the inner copper cylinder is $2a$ and the diameter of the outer copper cylinder shell is $2b$. If d is the cylinder's length and the relative permittivity of the fluid is ϵ_r . Find the general relation between the fluid level h and the capacitance C (i.e., the function $C(h)$). [15%]

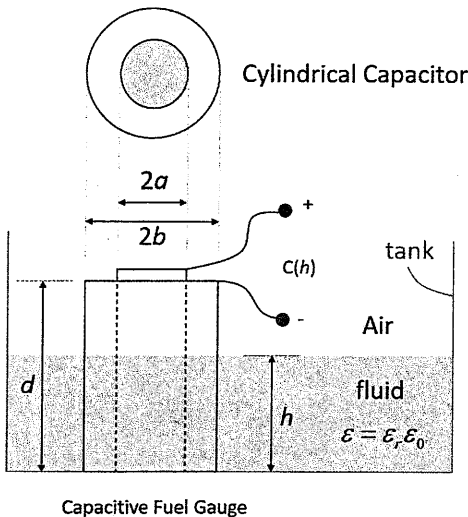
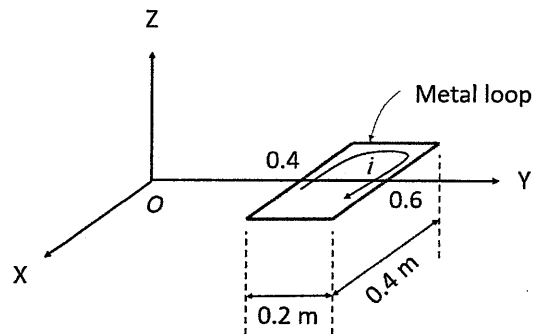


Fig. A



2. (a) The rectangular metal loop (with an internal series resistance of 10Ω) shown in Fig. B moves toward the original at a velocity of $\vec{v} = -100 \hat{a}_y$ (m/sec) in a magnetic flux field $\vec{B} = 0.5 e^{-0.5y} \hat{a}_z$ (T). Find the current at the instant when the coil sides are at $y = 0.4$ m and 0.6 m. [10%]
 (b) If the above-mentioned metal loop moves with a velocity of $\vec{v} = +20 \hat{a}_x$ along the x-direction, find the current magnitude. [5%]

3. Given the following electric field and magnetic field that satisfy the source-free Maxwell's equations in free space,

$$\vec{E}(z) = 20\pi e^{j(10^8 t - \beta z)} \hat{a}_x \quad (\text{V/m}); \quad \vec{H}(z) = H_0 e^{j(\omega t - \beta z)} \hat{a}_y \quad (\text{A/m})$$

- (a) Please show out the equation set of source-free Maxwell's equations in \vec{E} and \vec{H} notations. [5%]
 (b) Please find the wave propagation direction, ω , H_0 , and β ($\beta > 0$). [5%]
 (c) Find the time-average propagation power density (W/m^2). [5%]

4. The magnetic circuit shown in Fig. C is cast-iron with a mean length $\ell_c = 0.5$ (m) and square cross-section area of 4×10^{-4} (m²). The air-gap length is $\ell_a = 0.002$ (m) and the coil contains 400 turns. Find the current I required to establish an air-gap flux of 80×10^{-6} (Wb). Assume there is no fringing effect. The B-H curve of cast-iron is also given in Fig. C. [15%]

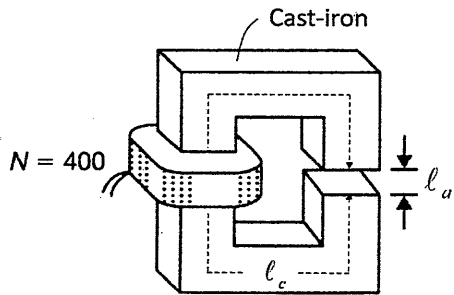
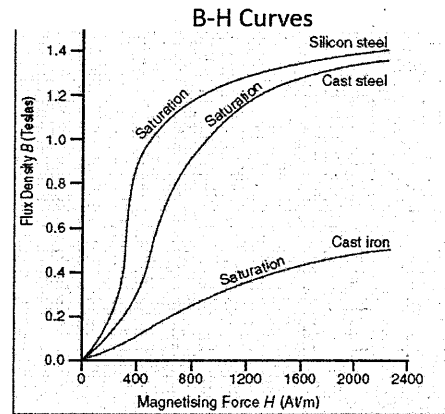


Fig. C



5. A lossless, air-filled cylindrical waveguide, which inside diameter is 3 cm, is operated at 14 GHz and placed along the z-axis. The solution of the electric field at a cross section in TM_{np} mode can be given by

$$E_{z,np}(r, \phi) = E_{np} J_n(k_{c,np} r) \cos(n\phi) \quad \text{where } n \in \{0, 1, 2, \dots\} \text{ and } p \in \{1, 2, 3, \dots\}$$

The roots of Bessel function $J_n(x)$ are given in the following table. For the TM_{21} mode propagating in the +z direction, find the cutoff frequency, guided wavelength, and wave impedance. [15%]

Zeros of $J_n(x)$, x_{np}

	n = 0	n = 1	n = 2
p = 1	2.405	3.832	5.136
p = 2	5.520	7.016	8.417
p = 3	8.645	10.173	11.620

6. (a) A plane wave incident on a plane dielectric boundary is illustrated in Fig. D. Please identify which case it belongs to. Perpendicular polarization or parallel polarization? Please also give your reason why. [2%]

(b) Given the incident electric and magnetic field intensity phasors in medium #1 as

$$\vec{E}_i(x,z) = \hat{a}_y E_{io} e^{-j\beta_1(x \sin\theta_i + z \cos\theta_i)} \quad \text{and} \quad \vec{H}_i(x,z) = \left(-\hat{a}_x \frac{E_{io}}{\eta_1} \cos\theta_i + \hat{a}_z \frac{E_{io}}{\eta_1} \sin\theta_i \right) e^{-j\beta_1(x \sin\theta_i + z \cos\theta_i)}$$

respectively. Please show out the reflected electric and magnetic field phasors in medium #1 and the transmitted electric and magnetic field phasors in medium #2. [8%]

(c) According to the boundary conditions of electromagnetic fields, please prove that the transmission coefficient τ

is equal to $\frac{2\eta_2 \cos\theta_i}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t}$. [5%]

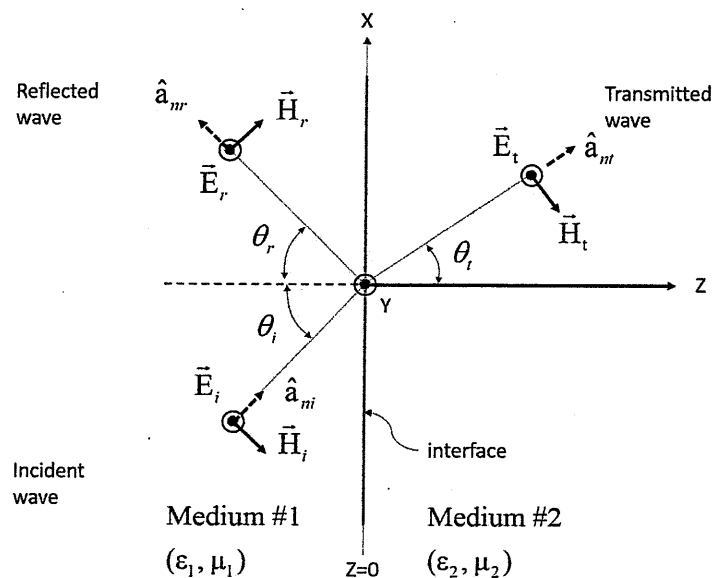


Fig. D

7. (a) There is an ideal directional antenna which radiates a total time-averaged power 1W in a solid angle of 0.2π uniformly. What is its directive gain? Note: You have to state your reason or you have to show your steps of calculation. [5%]

(b) A half-wavelength dipole antenna (whose directivity $D = 1.64$) is used to transmit at 80 MHz and 100 W. A receiving antenna in the form of a Hertzian dipole (whose directivity $D = 1.5$) is used to receive the wave power from the transmission antenna 20 km away. Please find the maximum power received by the receiving antenna. [5%]