

國立成功大學
110學年度碩士班招生考試試題

編 號：191

系 所：電腦與通信工程研究所

科 目：機率與線性代數

日 期：0203

節 次：第 3 節

備 註：不可使用計算機

※ 考生請注意：本試題不可使用計算機。請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. (15%) Let X and Y be independent (strictly positive) exponential random variables each with parameter λ . Are the random variables $X + Y$ and XY independent?
2. (10%) Let X and Y be jointly distributed with $\rho(X, Y) = 2/3$, $\text{Var}(X) = 1$, $\text{Var}(Y) = 9$. Find $\text{Var}(3X - 5Y + 7)$.
3. (15%) What is the probability of an even number of successes in n independent Bernoulli trials? (Bernoulli success probability is p)
4. (10%) Let X be a Poisson random variable with parameter λ . What is the maximum value of $\Pr(X)$ expressed by λ ?
5. (20%) Suppose that B is a 4×3 real-valued matrix. Let $S = BB^T$, and $T = B^T B$. Choose the true statement(s) from the following.
 - (a) It is possible that S is an invertible matrix.
 - (b) It is possible that T is an invertible matrix.
 - (c) Both S and T are diagonalizable.
 - (d) $I + S$ is an invertible matrix, where I is the identity matrix of the same size as S .
6. Consider two transformations T and S . The domain is the space of $n \times n$ real-valued matrices, while the codomain is the space of real numbers. Define $T(A) = \det(A)$, where $\det(A)$ is the determinant of A , and define $S(T) = \text{tr}(A)$, where $\text{tr}(A)$ is the trace of A .
 - (a) (10%) Is T a linear transformation? Is S a linear transformation?
 - (b) (10%) Is T a one-to-one transformation? Is S a one-to-one transformation?
 - (c) (10%) Is T an onto transformation? Is S a onto transformation?