

國立成功大學
110學年度碩士班招生考試試題

編 號： 183

系 所： 電腦與通信工程研究所

科 目： 通信數學

日 期： 0203

節 次： 第 3 節

備 註： 不可使用計算機

※ 考生請注意：本試題不可使用計算機。請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. (35%) A running integrator is defined as $y(t) = \int_{t-T}^t x(u)du$ where $x(t)$ is the input, $y(t)$ is the output, and T is the integration period. Both $x(t)$ and $y(t)$ are sample functions of stationary processes $X(t)$ and $Y(t)$, respectively.
 - (a) (10%) Express the power spectral density (PSD) of the integrator output, $S_Y(f)$, in terms of the PSD of the integrator input, $S_X(f)$. [10 points]
 - (b) (10%) If $X(t)$ is a white Gaussian process with $S_X(f) = \frac{N_0}{2}$, plot the PSD of $Y(t)$. Is $Y(t)$ a white process? Justify your answer.
 - (c) (15%) [continued from (b)] Find the probability density function (pdf) of Z where $Z = Y(3)$.
2. (15%) Let us assume that you are given $N = 10^6$ observations of a continuous random variable X . Describe and explain how you would go about to estimate the pdf of X .
3. (20%) Suppose that B is a 5×4 real-valued matrix. Let $S = BB^T$, and $T = B^T B$. Choose the true statement(s) from the following.
 - (a) It is possible that S is an invertible matrix.
 - (b) It is possible that T is an invertible matrix.
 - (c) Both S and T are diagonalizable.
 - (d) $I + S$ is an invertible matrix, where I is the identity matrix of the same size as S .
4. Consider two transformations T and S . The domain is the space of $n \times n$ real-valued matrices, while the co-domain is the space of real numbers. Define $T(A) = \det(A)$, where $\det(A)$ is the determinant of A , and define $S(T) = \text{tr}(A)$, where $\text{tr}(A)$ is the trace of A .
 - (a) (10%) Is T a linear transformation? Is S a linear transformation?
 - (b) (10%) Is T a one-to-one transformation? Is S a one-to-one transformation?
 - (c) (10%) Is T an onto transformation? Is S an onto transformation?