國立成功大學
110學年度碩士班招生考試試題

編　號： 184

系　所： 電腦與通信工程研究所

科　目： 電磁學及電磁波

日　期： 0203

節　次： 第 2 節

備　註： 可使用計算機

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考試科目：電磁學及電磁波

考試日期：0203，節次：2

※ 考生請注意：本試題可使用計算機。　請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

For your reference: $\varepsilon_0 = 10^{-9}/36\pi\ (F/m)$　$\mu_0 = 4\pi \times 10^{-7}\ (H/m)$　$\eta_0 = 120\pi\ (\Omega)$

Permitivity $\varepsilon = \varepsilon_r \varepsilon_0$　*Permeability* $\mu = \mu_r \mu_0$　*Conductivity* σ

1. A parallel-plate capacitor of width W, length L, and separation d is partially filled with a dielectric medium of dielectric constant ε_r, as shown in **Fig. A**. A battery with a voltage V_o is connected between the plates.

(a) Find the electric flux density $\vec{D}$ and the electric field intensity $\vec{E}$ in each region. [5%]

(b) Find the surface charge density on the top metal for each region. [5%]

(c) What is the field boundary condition that it should meet at the boundary A-A' ? [5%]

(d) Find the distance x such that the electrostatic energy stored in each region is the same. [5%]

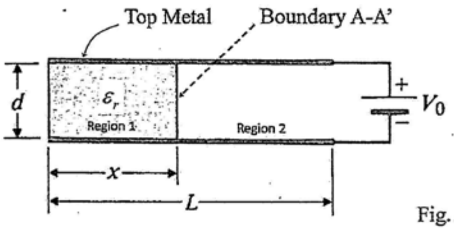

Fig. A

2. Determine the magnetic field intensity $\vec{H}$ for an infinitely-long solid cylindrical conductor of radius $r = 0.5$ m. The current density distribution $\vec{J}$ is given by

$$\vec{J}(r) = \begin{cases} 4e^{-2r}\,\hat{a}_z, & for\ \ 0 \le r \le 0.5\ \text{m} \\ 0, & for\ \ 0.5\ \text{m} < r \end{cases} \quad (\text{A/m}^2)$$

(a) Use Ampere's law to find $\vec{H}$ anywhere. [6%]

(b) Find the vector magnetic potential $\vec{A}$ in the region where $r > 0.5$ m. Assumed $|\vec{A}| = 0$ at $r = r_0$ where $r_0 >> 1$. [9%]

3. Given Laplace's equation of the potential function $V(r,\theta,\phi)$ in the spherical coordinates is

$$\nabla^2 V = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 \frac{\partial V}{\partial r}) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}(\sin\theta \frac{\partial V}{\partial \theta}) + \frac{1}{r^2 \sin^2\theta}\frac{\partial^2 V}{\partial \phi^2} = 0.$$

(a) Solve the Laplace's equation of $V(r,\theta,\phi)$ for the region between coaxial cones, as shown in **Fig. B1**. A potential V_1 is assumed at θ_1, and $V_2 = 0$ at θ_2. The cone vertices are insulated at $r = 0$ and the conducting cones are of infinite extent. (Please note: the solving procedures are required.) [10%]

(b) Find the capacitance between the two cones as shown in **Fig. B2**. Assumed in free space with $\varepsilon = \varepsilon_0$. [10%]

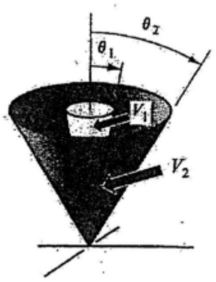

Fig. B1

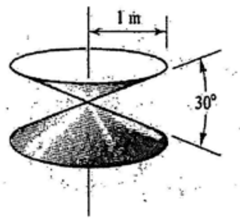

Fig. B2

4. The equivalent circuit of a differential length Δz of a two-conductor transmission line is shown in **Fig. C**. Let $v(z,t) = \text{Re}[V(z)\,e^{j\omega t}]$ and $i(z,t) = \text{Re}[I(z)\,e^{j\omega t}]$ where $V(z) = V_0^+ e^{-rz} + V_0^- e^{+rz}$; $I(z) = I_0^+ e^{-rz} + I_0^- e^{+rz}$; and $V_0^+, V_0^-, I_0^+, I_0^-$ are constant coefficients.

(a) Find the general transmission line equations for $v(z,t)$ and $i(z,t)$, respectively. [5%]

(b) Prove that the propagation constant r is equal to $\sqrt{(R+j\omega L)(G+j\omega C)}$. [5%]

(c) Prove that the characteristic impedance of the transmission line Z_0 is equal to $\sqrt{\dfrac{R+j\omega L}{G+j\omega C}}$. [5%]

(d) Find the parameters r, Z_0, and the phase velocity of a distortionless transmission line. [5%]

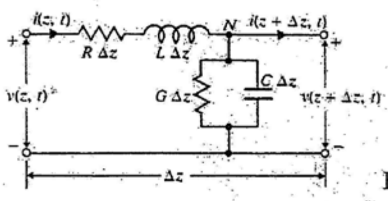

Fig. C

5. The phasors of the field components for TM_{mnp} modes in an air-filled rectangular cavity resonator (as shown in **Fig.D**) are given below.

$$E_x(x,y,z) = -\frac{E_0}{h^2}\left(\frac{m\pi}{a}\right)\left(\frac{p\pi}{d}\right)\cos\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{b}y\right)\sin\left(\frac{p\pi}{d}z\right),$$
$$E_y(x,y,z) = \ldots \quad unclear$$
$$E_z(x,y,z) = E_0\sin\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{b}y\right)\cos\left(\frac{p\pi}{d}z\right),$$
$$H_x(x,y,z) = \frac{j\omega\varepsilon_0 E_0}{h^2}\left(\frac{n\pi}{b}\right)\sin\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right)\cos\left(\frac{p\pi}{d}z\right),$$
$$H_y(x,y,z) = -\frac{j\omega\varepsilon_0 E_0}{h^2}\left(\frac{m\pi}{a}\right)\cos\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{b}y\right)\cos\left(\frac{p\pi}{d}z\right),$$
$$H_z(x,y,z) = 0,$$
$$where \quad h^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2;\ m,n \in N;\ p \in N \cup \{0\}.$$

And $a, b,$ and d are the dimensions in the *x*-, *y*-, and *z*-directions, respectively.

(a) Unfortunately, the formula expression of $E_y(x,y,z)$ is lost. Please rebuild the expression of $E_y(x,y,z)$ by way of the concept of wave impedance. [10%]

(b) If $a > b > d$, what is the dominant mode of this cavity resonator? And what is the resonant frequency? [5%]

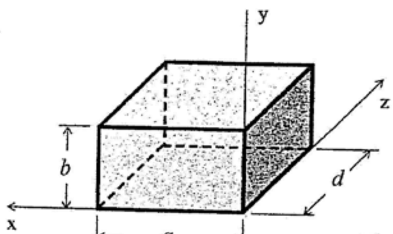

Fig. D

6. The far-zone fields of a Hertzian dipole ($I\,d\ell$) is given by

$$\begin{cases} \vec{H}_\phi = \hat{a}_\phi\, j\dfrac{I\,d\ell}{4\pi}\cdot\dfrac{e^{-j\beta R}}{R}\cdot\beta\sin\theta & \text{(A/m)} \\ \vec{E}_\theta = \hat{a}_\theta\, j\dfrac{I\,d\ell}{4\pi}\cdot\dfrac{e^{-j\beta R}}{R}\cdot\eta_0\beta\sin\theta & \text{(V/m)} \end{cases}$$

in the spherical coordinates. β is the phase constant and η_0 is the intrinsic impedance of air.

(a) Find the directive gain and the directivity of such a Hertzian dipole. [6%]

(b) Find the radiation resistance. [4%]