國立成功大學

111學年度碩士班招生考試試題

編　號： 191

系　所： 電腦與通信工程研究所

科　目： 機率與線性代數

日　期： 0219

節　次： 第 3 節

備　註： 不可使用計算機

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※ 考生請注意：本試題不可使用計算機。　請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. (10%) What is the probability that the sixth toss of a die is the first 6?

2. (15%) A woman and her husband want to have a 95% chance for at least one boy and at least one girl. What is the minimum number of children that they should plan to have? Assume that the events that a child is a girl and a boy are equiprobable and independent of the gender of other children born in the family.

3. (10%) Let X and Y be two independent uniformly distributed random variables over the intervals $(0,1)$ and $(0,2)$, respectively. Find the probability function of X/Y.

4. (15%) Let X and Y be independent random variables with common probability density function $f(x) = \begin{cases} e^{-x}, & x > 0 \\ 0, & \text{otherwise} \end{cases}$. Find the joint probability density function of $U = X + Y$ and $V = X - Y$.

5. (20%) Mark each of the following statements True (T) or False (F). (Need not to give reasons.)

   (a) If A is a square matrix, then the matrix $I + A$ must be an invertible matrix, where I is the identity matrix of the same size as A.

   (b) Let T be a linear operator on a vector space V. Then T^2 is also a linear operator.

   (c) Let W_1 and W_2 be two subspaces of a vector space V. then $W_1 \cup W_2$ is also a subspace V.

   (d) For an $n \times n$ matrix M, we have $\text{rank}(M^2) \leq \text{rank}(M)$.

6. (20%) Let B be a 4×4 matrix with eigenvalues $-1, 0, 1, 2$.

   (1) Find the determinant of $B^2 + 2I$, where I is the identity matrix of the same size as B.

   (2) Choose the invertible matrix (matrices) from the following. (a) B (b) $B+I$ (c) $B-I$ (d) B^2+I (e) $2B+I$

7. (10%) Let A be an $n \times n$ matrix. Consider the set $S = \{I, A, A^2, \ldots, A^n\}$, where I is the identity matrix of the ssame size as A. Determine if it is possible that S is a linearly independent set. You need to give the reason of your answer. (Hint: Cayley Hamilton)