

國立成功大學

111學年度碩士班招生考試試題

編 號： 183

系 所： 電腦與通信工程研究所

科 目： 通信數學

日 期： 0219

節 次： 第 3 節

備 註： 不可使用計算機

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※ 考生請注意：本試題不可使用計算機。請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. (30%) For a constant  $a > 0$ , random variables  $X$  and  $Y$  have joint probability density function

$$f_{X,Y}(x,y) = \begin{cases} \frac{1}{a^2}, & 0 \leq x, y \leq a \\ 0, & \text{otherwise} \end{cases}$$

Find the cumulative distribution function and probability density function of random variable

$$W = \max\left(\frac{X}{Y}, \frac{Y}{X}\right).$$

2. (20%) Let  $Z$  be the standard normal random variable, and  $Y$  is defined as  $Y = a + bZ + cZ^2$ . Find the correlation coefficient of  $Y$  and  $Z$  in terms of  $a$ ,  $b$ , and  $c$ .
3. (20%) Mark each of the following statements True (T) or False (F). (Need not to give reasons.)
- (a) Let  $T$  be a linear operator on a vector space  $V$ . Then  $T^2$  is also a linear operator.
  - (b) If  $A$  is a square matrix of size  $n$ , then the matrix  $I + A$  must be an invertible matrix, where  $I$  is the identity matrix of the same size as  $A$ .
  - (c) Let  $W_1$  and  $W_2$  be two subspaces of a vector space  $V$ . then  $W_1 \cup W_2$  is also a subspace  $V$ .
  - (d) For an  $n \times n$  matrix  $M$ , we have  $\text{rank}(M^2) \leq \text{rank}(M)$ .
4. (20%) Let  $B$  be a  $4 \times 4$  matrix with eigenvalues  $-1, 0, 1, 2$ .
- (1) Find the determinant of  $B^2 + 2I$ , where  $I$  is the identity matrix of the same size as  $B$ .
  - (2) Choose the invertible matrix (matrices) from the following. (a)  $B$  (b)  $B+I$  (c)  $B-I$  (d)  $B^2+I$  (e)  $2B+I$
5. (10%) Let  $A$  be an  $n \times n$  matrix. Consider the set  $S = \{I, A, A^2, \dots, A^n\}$ , where  $I$  is the identity matrix of the same size as  $A$ . Determine if it is possible that  $S$  is a linearly independent set. You need to give the reason of your answer. (Hint: Cayley Hamilton)