國立成功大學
111學年度碩士班招生考試試題

編　號：185

系　所：電腦與通信工程研究所

科　目：電磁數學

日　期：0219

節　次：第 3 節

備　註：不可使用計算機

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※ 考生請注意：本試題不可使用計算機。　請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. (10%) Find all solutions for z that $e^z = 1 + i$.

2. (20%) Find the eigenvalues and eigenfunctions of the equation.

$$y'' + 3y' + 2y + \lambda y = 0,$$
$$y(0) = 0,\ y(1) = 0.$$

3. (20%) Find the coefficients a_0, a_1, a_2, a_3, and a_4 in the series solution

$$y = \sum_{n=0}^{\infty} a_n x^n$$

for the equation

$$y'' + 3xy' + (4 + 2x^2)y = 0,$$
$$y(0) = 2,\ y'(0) = -3.$$

4. (20%) Mark each of the following statements True (T) or False (F). (Need not to give reasons.)

   (a) If A is a square matrix of size n, then the matrix $I + A$ must be an invertible matrix, where I is the identity matrix of the same size as A.

   (b) Let T be a linear operator on a vector space V. Then T^2 is also a linear operator.

   (c) Let W_1 and W_2 be two subspaces of a vector space V. then $W_1 \cup W_2$ is also a subspace V.

   (d) For an $n \times n$ matrix M, we have $\text{rank}(M^2) \leq \text{rank}(M)$.

5. (20%) Let B be a 4×4 matrix with eigenvalues $-1, 0, 1, 2$.

   (1) Find the determinant of $B^2 + 2I$, where I is the identity matrix of the same size as B.

   (2) Choose the invertible matrix (matrices) from the following. (a) B (b) $B+I$ (c) $B-I$ (d) $B^2 + I$ (e) $2B + I$

6. (10%) Let A be an $n \times n$ matrix. Consider the set $S = \{I, A, A^2, \ldots, A^n\}$, where I is the identity matrix of the same size as A. Determine if it is possible that S is a linearly independent set. You need to give the reason of your answer. (Hint: Cayley Hamilton)