

國立成功大學

112學年度碩士班招生考試試題

編 號： 182

系 所： 電腦與通信工程研究所

科 目： 通信數學

日 期： 0206

節 次： 第 1 節

備 註： 不可使用計算機

※ 考生請注意：本試題不可使用計算機。請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. (35%) Random variables X and Y have joint probability density function (PDF)

$$f_{X,Y}(x,y) = \begin{cases} \frac{5x^2}{2}, & -1 \leq x \leq 1, 0 \leq y \leq x^2, \\ 0, & \text{otherwise.} \end{cases}$$

Let $A = \{Y \leq \frac{1}{4}\}$.

- Find the conditional PDF $f_{X,Y|A}(x,y)$.
 - Find the conditional PDF $f_{Y|A}(y)$ and $E[Y|A]$.
 - Find the conditional PDF $f_{X|A}(x)$ and $E[X|A]$.
2. (15%) Let X be a random variable such that $E[X^{2k}] = \frac{(2k)!}{k!}$ and $E[X^{2k+1}] = 0$, $k = 0, 1, 2, \dots, \infty$. Find the moment generating function of X . What is the PDF of X ?
3. (20%) Mark each of the following statements True (T) or False (F). (Need not to give reasons.)
- Let A be an invertible matrix. Then $A + A^2$ is also an invertible matrix.
 - For an invertible matrix M , we have $\text{rank}(M^2) = \text{rank}(M)$.
 - If A is a real-valued invertible square matrix of size n , then A^2 is also an invertible matrix.
 - If three $n \times n$ matrices A , B , and C satisfy $AB = AC$, then we have $B = C$.
4. (20%) Consider a linear transformation T on \mathbb{R}^3 , define by

$$T\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} x \\ x+y \\ x+y+z \end{bmatrix}$$

Find the standard matrix of T . Also, find the inverse of T . (Express your answer as

$$T^{-1}\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \dots)$$

5. (10%) Suppose the characteristic polynomial of a 4×4 matrix M is $p(t) = 2t^4 - t^3 + 7t^2 - 3t + 5$. Find the determinant of M .