

國立成功大學

113學年度碩士班招生考試試題

編 號：185

系 所：電腦與通信工程研究所

科 目：機率與線性代數

日 期：0201

節 次：第 3 節

備 註：不可使用計算機

※ 考生請注意：本試題不可使用計算機。請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. (30%) Two coins are simultaneously tossed until one of them comes up a head and the other a tail. The first coin comes up a head with probability  $p$ , and the second coin comes up a head with probability  $q$ . All tosses are assumed independent.
- (a) Find the probability mass function (PMF) of the number of tosses. (10%)
- (b) Find the expected value of the number of tosses. (10%)
- (c) What is the probability that the last toss of the first coin is a head? (10%)

2. (20%) A dart is equally likely to land at any point inside a circular target of radius 4. Let  $R$  be the distance of the landing point from the origin.
- (a) Find and plot the probability density function (pdf) of  $R$ . (10%)
- (b) The 'bull eye' is the central disk in the target of radius 1.

Find the probability that the dart is outside of the bull eye. (10%)

Hint: Let  $S$  be a subset of the plane with area  $A(S)$  [which means "area of  $S$ "]. A point is said to be randomly selected from  $S$  if, for any subset  $T$  of  $S$  with area  $A(T)$ , the probability that the point selected falls in  $T$  is  $\frac{A(T)}{A(S)}$ , i.e., the ratio of "area of  $T$ " to "area of  $S$ ".

3. (20%) Mark each of the following statements True (T) or False (F). (Need not to give reasons.)
- (a) If the column vectors of a matrix  $A$  are linearly independent, then the row vectors of  $A$  are also linearly independent.
- (b) Let  $S$  be a linearly independent set in a vector space  $V$ . If  $\mathbf{u}$  is a vector in  $V$  and  $\mathbf{u} \notin S$ , then  $S \cup \mathbf{u}$  is also a linearly independent set.
- (c) Let  $T$  and  $S$  be two linear transformations (operations) on a vector space  $V$ , then  $T^2 + 2S$  is also a linear transformation on  $V$ .
- (d) Let  $M$  be a square matrix. We have that  $M$  and  $M^2$  must be linearly independent.

4. (30%) Consider a linear transformation  $T$  on  $\mathbb{R}^3$ , define by  $T\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} x + y \\ y + z \\ z + x \end{bmatrix}$ .
- Find the standard matrix of  $T$ . Also, find the inverse of  $T$ . (Express your answer as  $T^{-1}\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \dots$ ). What is the rank of  $T$ ?